CS261: Exercise Set #4

For the week of April 20–24, 2015

Instructions:

(1) *Do not turn anything in.*

(2) The course staff is happy to discuss the solutions of these exercises with you in office hours or on Piazza.

(3) While these exercises are certainly not trivial, you should be able to complete them on your own (perhaps after consulting with the course staff or a friend for hints).

Exercise 16

Recall that in the maximum-weight bipartite matching problem, the input is a bipartite graph $G = (A, B, E)$ with a nonnegative weight w_e per edge, and the goal is to compute a matching M that maximizes $\sum_{e \in M} w_e$.

In the minimum-cost perfect bipartite matching problem, the input is a bipartite graph $G = (A, B, E)$ such that $|A| = |B|$ and G contains a perfect matching, and a nonnegative cost c_e per edge, and the goal is to compute a perfect matching M that minimizes $\sum_{e \in M} c_e$.

Give a linear-time reduction from the former problem to the latter problem.

Exercise 17

In the *edge cover* problem, the input is a graph $G = (V, E)$ (not necessarily bipartite) with no isolated vertices, and the goal is to compute a minimum-cardinality subset $F \subseteq E$ of edges such every vertex $v \in V$ is the endpoint of at least one edge in F. Prove that this problem reduces to the maximum-cardinality matching problem.

Exercise 18

Encode the maximum flow problem as a linear program. The size of your linear program should be linear in that of the maximum flow instance.

Exercise 19

Prove that solving a linear program of the form

$$\min \mathbf{c}^T \mathbf{x}$$

subject to

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0$$

reduces in polynomial time to solving a linear program of the form discussed in lecture:

$$\max \mathbf{c}^T \mathbf{x}$$

subject to

$$\mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0.$$

Exercise 20

Consider a linear program of the form

$$\min \mathbf{c}^T \mathbf{x}$$

subject to

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0,$$

and call this the "primal" linear program. Define the "dual" linear program as

$$\max \mathbf{b}^T \mathbf{y}$$

subject to

$$\mathbf{A}^T \mathbf{y} \leq \mathbf{c},$$

where the decision variables $\mathbf{y}$ are unrestricted. State and prove a "weak duality" result for this primal-dual pair of linear programs.