CS364B: Problem Set #1

Two problems (your choice) due in class on Monday, October 8, 2007.
Final problem due in class on Monday, October 15, 2007.

Instructions:

This problem set contains three problems. Hand in your solution to two of them (your choice) by October 8th, and your solution to the third problem by October 15th.

Collaboration on this homework is actively encouraged. However, your write-up must be your own, and you must list the names of your collaborators on the front page.

Keep an eye on the course web site for a FAQ on this homework.

Problem 1

Consider the following keyword auction setting. There are n bidders with value-per-click $v_1, v_2, v_3 \dots v_n$. There are k slots with click-through-rates (CTRs) $\Theta_1 \geq \Theta_2 \geq \cdots \geq \Theta_k$. We assume that the CTR of advertiser i's advertisement in slot j is Θ_j. Assume that $k > n$. (This is without loss of generality; if the condition is not satisfied, we can always add dummy slots with CTR 0.)

(a) In the first lecture we saw that the GSP is not truthful when there are two identical slots. In practice higher slots get more clicks. It is more realistic to assume that CTRs fall geometrically; suppose $\Theta_j = 2^{-j}$ for all j. Demonstrate by example that GSP is still not truthful.

(b) Prove that over-bidding (bidding higher than one's value) is never beneficial in GSP.

(c) Call a keyword-auction *monotone* if it has the following property for every advertiser i and every fixed set b_{-i} of bids by the other advertisers: If advertiser i is allocated slot j when it bids b_i, it is allocated slot $j' \leq j$ (a slot with a higher CTR) if it bids $b'_i \geq b_i$. Prove that every truthful keyword-auction is monotone. (Assume that $\Theta_1 > \Theta_2 > \cdots > \Theta_k$.)

Also, is GSP monotone?

Problem 2

We continue in the setup of the first problem. Motivated by the discussion in the October 1 lecture, consider the following auction: (1) collect bids and rank advertisers in order of bids (relabel the bidders so that the ith bidder receives the ith slot); (2) charge the ith bidder a total payment P_i equal to its externality (using other advertisers' bids as proxies for their valuations):

$$P_i = \sum_{j>i} (\Theta_{j-1} - \Theta_j) b_j,$$

or equivalently, a price per click p_i equal to

$$p_i = \sum_{j>i} \left(\frac{\Theta_{j-1} - \Theta_j}{\Theta_i} \right) b_j.$$

For historical reasons, call this the *VCG auction.*

(a) Prove that the VCG auction is truthful.

(b) Prove that the VCG auction is the only auction that: (1) is truthful; (2) ranks advertisers by bid; and (3) charges the last advertiser zero per click. (I.e., if you rank by bid, no other pricing scheme yields a truthful auction.)

[Hint: First argue that for all i,

$$\Theta_i \cdot p_i - \Theta_{i+1} \cdot p_{i+1} \geq (\Theta_i - \Theta_{i+1}) \cdot b_{i+1}$$

and

$$\Theta_i \cdot p_i - \Theta_{i+1} \cdot p_{i+1} \leq (\Theta_i - \Theta_{i+1}) \cdot b_{i+1}$$

Use this to complete the proof.]

(c) For each of the following allocation rules, determine whether or not it can be extended into a truthful auction via suitable payments. In either case, formally prove your answer.

(i) Reject all advertisers with bids less than a reserve price r. Rank the rest by bid.

(ii) Reject the highest bidder. Rank the rest by bid.

Problem 3

(a) Exercise 28.2 from the Lahaie/Pennock/Saberi/Vohra book chapter.

(b) **(Extra credit.)** Exercise 28.3 from the Lahaie/Pennock/Saberi/Vohra book chapter.