CS364A: Exercise Set #3

Due by the beginning of class on Wednesday, October 16, 2013

Instructions:

(1) Turn in your solutions to all of the following exercises directly to one of the TAs (Kostas or Okke). Please type your solutions if possible and feel free to use the LaTeX template provided on the course home page. Email your solutions to `cs364a-aut1314-submissions@cs.stanford.edu`. If you prefer to hand-write your solutions, you can give it to one of the TAs in person at the start of the lecture.

(2) Your solutions will be graded on a "check/minus/zero" system, with "check" meaning satisfactory and "minus" meaning needs improvement.

(3) Solve these exercises and write up your solutions on your own. You may, however, discuss the exercises verbally at a high level with other students. You may also consult any books, papers, and Internet resources that you wish. And of course, you are encouraged to contact the course staff (via Piazza or office hours) to clarify the questions and the course material.

(4) No late assignments will be accepted.

Lecture 5 Exercises

Exercise 20

Consider a single-item auction with two bidders with valuations drawn i.i.d. from the uniform distribution on $[0,1]$.

(a) Prove that the expected revenue obtained by the Vickrey auction (with no reserve) is $\frac{1}{3}$.

(b) Prove that the expected revenue obtained by the Vickrey auction with reserve $\frac{1}{2}$ is $\frac{5}{12}$.

Exercise 21

Compute the virtual valuation function of the following distributions.

(a) The uniform distribution on $[0,a]$ with $a > 0$.

(b) The exponential distribution with rate $\lambda > 0$ (on $[0,\infty)$).

(c) The distribution given by $F(v) = 1 - \frac{1}{(v+1)^c}$ on $[0,\infty)$, where $c > 0$ is some constant.

Which of these distributions are regular (meaning the virtual valuation function is strictly increasing)?

Exercise 22

Consider the distribution in part (c) of the previous problem, with $c = 1$. Argue that when bidder valuations are drawn from this distribution, it is *not* necessarily the case that the expected revenue of an auction equals its expected virtual surplus. To reconcile this observation with the main result from Lecture 5, identify which assumption from lecture is violated in this example.

Exercise 23

Consider an auction with k identical goods, with at most one given to each bidder. There are n bidders whose valuations are i.i.d. draws from a regular distribution F. Describe the optimal auction in this case. Which of the following does the reserve price depend on: k, n, and/or F?

Exercise 24

Repeat the previous exercise for sponsored search auctions, with n bidders with valuations-per-click drawn i.i.d. from a regular distribution, and with $k \leq n$ slots with $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_k$.

Lecture 6 Exercises

Exercise 25

Consider an arbitrary single-parameter environment, with feasible set X. Suppose bidder i's valuation is drawn from a regular distribution F_i, with strictly increasing virtual valuation function φ_i. The virtual surplus-maximizing allocation rule is $\mathbf{x}(\mathbf{b}) = \arg\max_{(x_1,\ldots,x_n)\in X} \sum_{i=1}^{n} \varphi_i(b_i)x_i$. Prove that this allocation rule is monotone.

[You should assume that ties are broken in a deterministic and consistent way, such as lexicographically.]

Exercise 26

Consider a single-item auction where bidder i's valuation is drawn from its own regular distribution F_i (i.e., the F_i's can be different).

(a) Give a formula for the winner's payment in an optimal auction, in terms of the bidders' virtual valuation functions.

(b) Show by example that, in an optimal auction, the highest bidder need not win, even if it has a positive virtual valuation. [Hint: two bidders with valuations from different uniform distributions suffices.]

(c) Give an intuitive explanation of why the property in (b) might be beneficial to the revenue of an auction.

Exercise 27

Consider a single-item auction with n bidders with valuations drawn i.i.d. from a regular distribution F. Prove that the expected revenue of the Vickrey auction (with no reserve) is at least $\frac{n-1}{n}$ times that of the optimal auction (with the same number n of bidders).

[Hint: deduce this statement from the Bulow-Klemperer theorem. When one new bidder is added, how much can the maximum-possible expected revenue increase?]