

# CS269I: Exercise Set #4

Due by 11:59 PM on Wednesday, October 24, 2018

## Instructions:

- (1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.
- (2) Submission instructions: We are using Gradescope for the homework submissions. Go to [www.gradescope.com](http://www.gradescope.com) to either login or create a new account. Use the course code MZZ2BV to register for CS269I. Only one person needs to submit the assignment. When submitting, please remember to add your partner's name (if any) in Gradescope.
- (3) Please type your solutions if possible. We encourage you to use the LaTeX template provided on the course home page.
- (4) Write convincingly but not excessively. You should be able to fit all of your solutions into two pages, if not less.
- (5) Except where otherwise noted, you may refer to the course lecture notes and the specific supplementary readings listed on the course Web page *only*.
- (6) You can discuss the exercises verbally at a high level with other groups. And of course, you are encouraged to contact the course staff (via Piazza or office hours) for additional help.
- (7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.
- (8) No late assignments will be accepted, but we will drop your lowest exercise set score.

## Lecture 7 Exercises

### Exercise 14

Consider a single-item auction with at least three bidders. Prove that awarding the item to the highest bidder, at a price equal to the third-highest bid, yields an auction that is *not* truthful.

### Exercise 15

Prove that for every false bid  $b_i \neq v_i$  by a bidder in a second-price auction, there exist bids  $\{b_j\}_{j \neq i}$  by the other bidders such that  $i$ 's utility when bidding  $b_i$  is strictly less than when bidding  $v_i$ .

### Exercise 16

Suppose there are  $k$  identical copies of an item and  $n > k$  bidders. Suppose also that each bidder can receive at most one item. What is the analog of the second-price auction? Prove that your auction is truthful.

## Lecture 8 Exercises

### Exercise 17

Show that the general VCG mechanism is “individually rational,” meaning that a truthful bidder is guaranteed nonnegative utility.<sup>1</sup> [Hint: prove that  $p_i \leq b_i(\omega^*)$ , where  $p_i$  is the VCG payment by bidder  $i$ ,  $\omega^*$  is the outcome chosen by the mechanism, and  $b_i(\omega^*)$  is the bid by bidder  $i$  for the outcome  $\omega^*$ .]

### Exercise 18

- (a) Prove that, for every fixed  $r \geq 0$ , a Vickrey single-item auction with reserve price  $r$  is truthful.
- (b) Propose how to incorporate a reserve price into the VCG sponsored search auction while preserving the auction’s truthfulness. Prove that your proposal is truthful.

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<sup>1</sup>You can assume that all bids are nonnegative.