

Bonus Lecture #1: The FLP Impossibility Theorem

COMS 4995-001:
The Science of Blockchains
URL: <https://timroughgarden.org/s25/>

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Goals for Bonus Lecture #1

1. Understanding the asynchronous model.

- what does “no assumptions on message delays” mean?

2. Proof of the FLP Theorem.

- state machine replication (SMR) is “unsolvable” in asynchrony
- need to compromise to make further progress
 - pull back to “partial synchrony” (see next lecture)
 - relax consistency guarantees (could be a good project)
 - randomized protocols that succeed with high probability
 - could also be a good project

SMR: Synchrony vs. Asynchrony

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Question: what's the “asynchronous model”?

The Setup (\approx Asynchronous Model)

- shared global clock, timesteps $0, 1, 2, \dots$
 - traditional asynchronous model does not have this (only makes today's impossibility result stronger)
- pool M of outstanding messages (sent but not yet received)

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 2. non-crashed validators decide which txs to finalize, messages to send
 - as instructed by whatever protocol they're running
 - messages sent injected directly into M

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Theorem: [FLP85] no SMR protocol guarantees consistency and liveness in the setup above.

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- “protocol” = specifies what validators should do in each timestep
 - as a function of their input, the timestep, and messages received
- think of Π as deterministic (or with adversary-controlled randomness)

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1. liveness of $\Pi \rightarrow$ every non-faulty validator eventually outputs 0 or 1
2. consistency of $\Pi \rightarrow$ all non-faulty validators eventually output the same thing
3. if all inputs are 0 (respectively, 1) \rightarrow all outputs are 0 (respectively, 1)

Configurations

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Proof plan: devise strategy of adversary resulting in an infinite sequence $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow \dots$ of configurations such that no validator outputs in any C_t . [note: would contradict liveness]

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 - (technically, defined only for configurations C with at most one crash)

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Next: define a “pivotal” configuration as (roughly) one in which crashing a validator flips the output of the protocol.

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 - as far as other validators $j \neq i$ can tell, i crashed at time t'
 - only difference is the state of M , which validators do not observe

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Definition: for an i -restricted configuration C , $val(C \setminus i) :=$ output of the protocol Π with an adversary that:

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- **key point:** C pivotal \rightarrow no validators have output yet (why?)

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Recall proof plan: devise strategy of adversary resulting in an infinite sequence $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow \dots$ of configurations such that no validator outputs in any C_t . [contradicts liveness]

- suffices to use only pivotal configurations
- we will exhibit such a sequence, inductively

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- let X_i = initial configuration in which validators $1, 2, \dots, i$ have input 1 and validators $i+1, i+2, \dots, n$ have input 0
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 - **in general:** if validator sees identical messages at every timestep in two different executions, will behave identically (including the same output)

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 - if i crashes immediately, doesn't matter whether its input was 0 or 1
- **so:** either (i) $\text{val}(X_{i-1} \setminus i) = 1$ (in which case X_{i-1} is i -pivotal) or (ii) $\text{val}(X_i \setminus i) = 0$ (in which case X_i is i -pivotal)
 - either way, we have our initial pivotal configuration C_0

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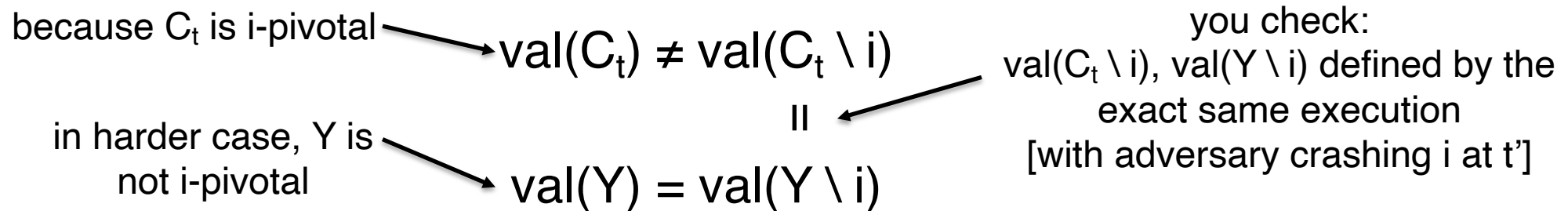
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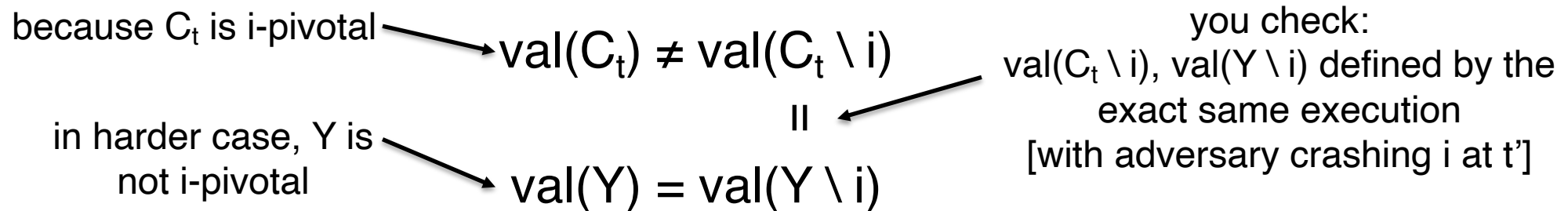
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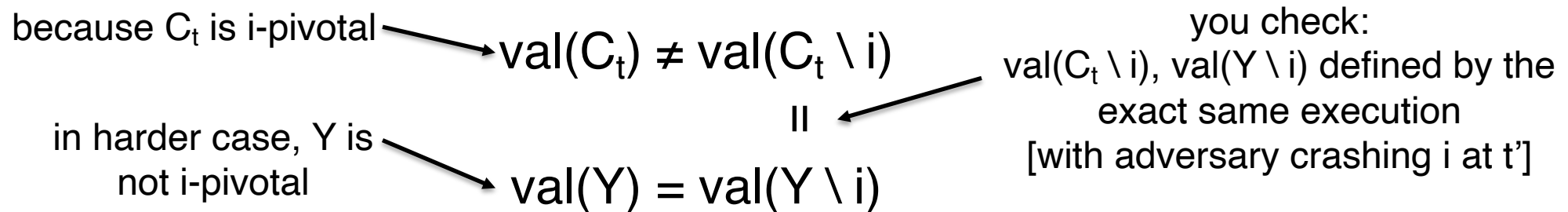
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- **on the other hand:** $\text{val}(Y_{r-1} \setminus j) = \text{val}(Y_r \setminus j)$
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- **so:** either (i) $\text{val}(Y_{r-1} \setminus j) = 1$ (in which case Y_{r-1} is j -pivotal) or (ii) $\text{val}(Y_r \setminus j) = 0$ (in which case Y_r is j -pivotal)
 - either way, we have our next pivotal configuration C_{t+1}

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Upshot: there is an infinite sequence $C_0 \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \rightarrow \dots$ of pivotal configurations (\rightarrow no validator outputs in any C_t).

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 - **fix:** modify adversary strategy to crash i at timestep t , act benign thereafter
 - other validators can't tell the difference, protocol behavior unchanged
 - now a valid adversary strategy \rightarrow contradicts liveness of $\Pi!$

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3. Use randomized protocols, solve SMR with high probability.
 - rich academic literature on this topic