Bonus Lecture #3: Digital Signatures in Blockchain Protocols (Part 2 of 2)

COMS 4995-001: The Science of Blockchains

URL: https://timroughgarden.org/s25/

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Defining Digital Signature Schemes

Digital signature scheme: defined by 3 (efficient) algorithms:

1. *Key generation algorithm:* maps seed r → (pk,sk) pair.
   - in some cases, may generate r itself (e.g., ssh-keygen)
2. *Signing algorithm:* maps message + sk → signature.
   - signature depends on both sk and the message being signed
3. *Verification algorithm:* maps msg + sig + pk → "yes"/"no".
   - anyone who knows pk can verify correctness of an alleged signature

Goals for Bonus Lecture #3

1. Schnorr signatures.
   - used in Bitcoin since the Taproot upgrade in 2021, EdDSA in Solana
2. tl;dr of elliptic curves.
   - groups where discrete log appears harder than in Z_p^*
   - basis of all signature schemes used in blockchain protocols
3. tl;dr of ECDSA signatures.
   - what users use to sign transactions in Bitcoin and Ethereum
4. tl;dr of BLS signatures.
   - used by Ethereum validators to sign consensus-layer messages

Signatures Based on Exponentiation

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  - group: has a well-defined binary operation, each element has inverse
  - ex: each element of Z_7^* a power of 3, each element of Z_{11}^* a power of 2

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Discrete log (DL) assumption: there is no (randomized) polynomial-time algorithm that can recover x from g and g^x (with non-negligible probability). [necessary condition for security]

Computing Discrete Logarithms

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- solvable with $O((\ln q)^3)$ group operations on quantum computer
  – big quantum computers ➔ discrete log approach to signatures broken
- in Z_p^*, can solve with $\approx \exp\{1.92 \times (\ln p)^{1/3} \times (\ln \ln p)^{2/3}\}$ ops
  – for 128 bits of security, need ≥ 3072-bit private keys
  – same story for RSA signatures (can use GNFS for factoring, as well)

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2. can compute signatures without knowing sk
   - silly example: signature = message (or f(message))
3. signature leaks (computationally recoverable) info about sk
   - silly example: signature = sk
     - or anything from which sk can be easily extracted

Schnorr Signatures

Schnorr Signatures (in One Slide)

- let G = cyclic group with generator g, prime order $q \approx 2^t$
- key generation:
  - sk = random x in {0,1,2,…,q-1}
  - pk = g^x
- to sign:
  - choose random b in {0,1,2,…,q-1}
  - set a := $h(m \,||\, g^b)$ [h = cryptographic hash function, acts like random]
  - output as signature ($r:=g^b$, $s:=(ax+b) \bmod q$) [≈ 2t bits]
- to verify: accept signature (r,s) ⇔ $g^s = (pk)^{h(m \,||\, r)} \cdot r$

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- fix: use h(m) instead of m, where h = a cryptographic hash function

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Revised starting point: $f(x,m) := h(m) \cdot x \pmod q$. [h = CHF]

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  - insight: using that s = h(m) • x + b (mod q) ⇔ $g^s = g^{h(m) \cdot x+b}$, see that verification algorithm only needs to know g^b, not b itself

Deriving Schnorr Signatures (con'd)

Proposed signing algorithm: [m = message, x = private key, h = CHF]

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Proposed verification algorithm: [given m, pk, and (r,s)]

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- $h(m_1)=h(m_2)$ ➔ same signatures (r,s) valid for both m_1 and m_2

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- h CHF (as good as random) ➔ infeasible to find r satisfying $r = g^{-x \cdot h(m \,\|\, r)+s}$

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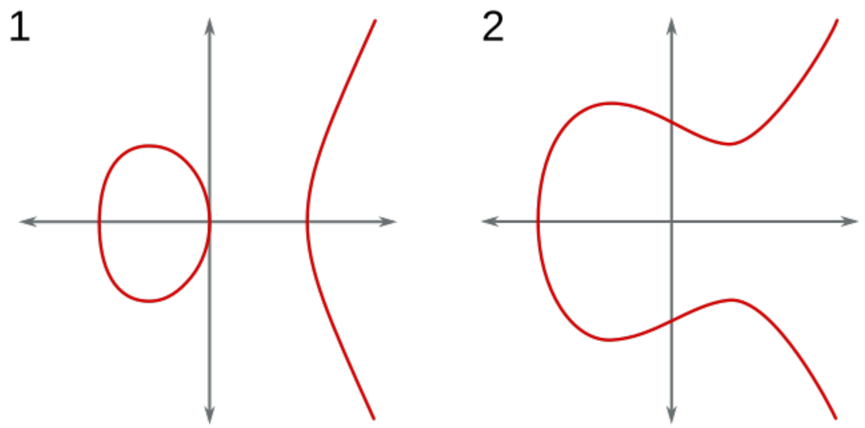
  - output as signature $(r:=g^b, s:=(ax+b) \bmod q)$ [$\approx$ 2t bits]
- to verify: accept signature $(r,s) \Leftrightarrow g^s = (pk)^{h(m \,||\, r)} \cdot r$

Elliptic Curves

Approximate definition: an *elliptic curve* is the set of solutions (x,y) to an equation of the form $y^2 = x^3+ax+b$ (for some a,b).



$y^2 = x^3 - x$ $\qquad$ $y^2 = x^3 - x + 1$

Elliptic Curves (over Finite Fields)

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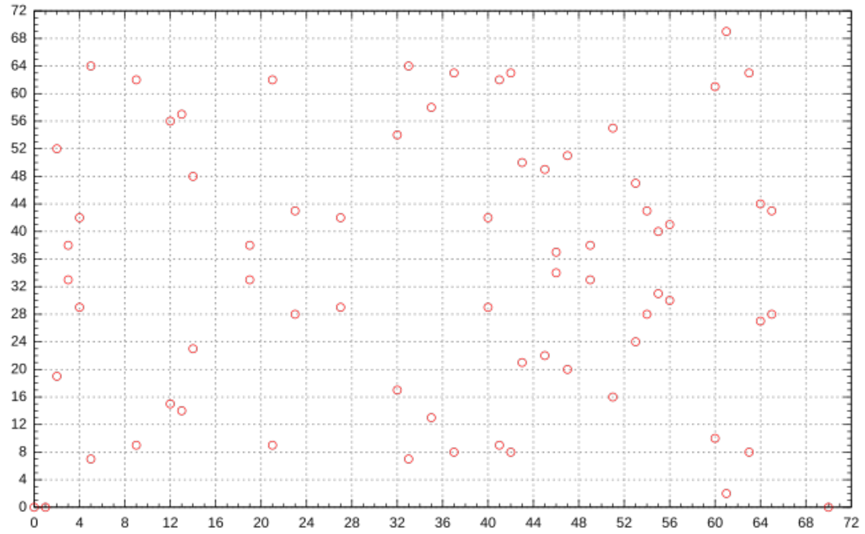

Set of affine points of elliptic curve $y^2 = x^3 - x$ over finite field $\mathbf{F}_{71}$.

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Example: secp256k1. [used in Bitcoin and Ethereum]

- defining equation: $y^2 = x^3 + 7 \bmod (2^{256}-2^{32}-977)$
- group of prime order (➔ cyclic), canonical generator

ECDSA Signatures

tl;dr: like Schnorr signatures, with two changes:

1. start from $f(x) = (x + h(m)) \bmod q$ rather than $f(x) = (h(m) \cdot x) \bmod q$
   – same chain of reasoning leads to a different verification equation (from ElGamal)

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- to sign: [some details omitted]
  – choose random a in $\{0,1,2,\ldots,q-1\}$, $r :=$ x-coordinate of g^a
  – set $s := a^{-1}(r \cdot x + h(m)) \bmod q$

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- to sign: [some details omitted]
  – choose random a in {0,1,2,…,q-1}, r := x-coordinate of g^a
  – set $s := a^{-1}(r \cdot x + h(m)) \bmod q$
- to verify: accept signature (r,s) ⇔ r = x-coordinate of $(g^{h(m)} \cdot (pk)^r)^{s^{-1}}$

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- to verify (m,pk,sig): accept $\Leftrightarrow$ $e(H(m),pk) = e(sig, g_2)$