Lecture #10: Cryptographic Hash Functions

> COMS 4995-001: The Science of Blockchains URL: https://timroughgarden.org/s25/

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Goals for Lecture #10

- 1. Short, unique names that require no coordination.
 - e.g., for transactions or blocks
- 2. Cryptographic hash functions as "random oracles."
 - ideal hash function = random function, though still has collisions
- 3. What do cryptographic hash functions actually look like?
 - case study: SHA-256 and length-extension attacks
- 4. Cryptographic commitments.
 - reconstructing blocks from hashes; binding and hiding

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Question: how should a block specify its predecessor?

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Question: how should a block specify its predecessor?

Ideally: use some "naming function" h(.) such that:

- h is easy to evaluate
- the output of h is short
- never have ambiguous/non-unique names: $x \neq x' \rightarrow f(x) \neq f(x')_{f}$

Definition: a hash function h maps each finite-length string x to an element h(x) of some range Y. [canonical example: $Y = \{0,1\}^{256}$]

- length of x can be as long as you want (e.g., text of *War and Peace*)

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Best-case scenario: a function h for which we'll never encounter a collision in practice (no matter how hard an adversary might try).

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Also: (more detailed but elementary probability calculations)

- # of evaluations << $2^{128} \rightarrow$ almost no chance of a collision
- # of evaluations >> $2^{128} \rightarrow$ almost no chance of no collisions

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Upshot: can use a collision-resistant hash function (with e.g. 256bit outputs) to provide objects with names that are short and (for all practical purposes) unique. [no coordination necessary!]

♥ Block 885092		f1349af59d8afe5c84289cfea2cea1a952dbf924d01050650c01e84d7	/cec7540	Details +
00000000000000000000000000000000000000		#0 e330bf80abfbc6934796b6316dc2167c2437ecef5b88e 0.10774214 BTC 4d0217a9186523fd550:0	#0 16TZSJLLWnwxqyUz5A17PC7X9n25GV8FwZ	1.03230000 BTC
		#1 05198522581a5f501063eb24dff4599365eadd889240 0.00528848 BTC 07f5c4512fdd760a836e:194		
		#2 7bc26b6c55756644e610d9d582f3427011398a03f4dc 0.16007236 BTC 8841a9afc898a83e59bc:10		
	Details +	#3 166b76eb7bf88c70fdaf0d91b0514d5bea3caeba73cc2 0.11054856 BTC 4eb60812590b2a9e2c0:0		
HEIGHT	885092	#4 13d29fda99fd051d4a05065ca8eee227e4674b46c578 0.21000000 BTC 3080ec5a9b087477ef46:0	>	
STATUS	In best chain (2 confirmations)	#5 a4d6acb47ec094968f11bb1db0aa20363c1f593cd0da 0.10998202 BTC 4626ffc91d643c6c8f0e:2		
TIMESTAMP	2025-02-24 04:29:55 GMT -5	#6 83224c4cd662adef9692ccfdbcbe6da9c43d5c6ec7ed 0.10770619 BTC 00cfdc71685629c84858:0		
SIZE	1784.384 KB	#7 298d91cc366f0ae82c59b149591be113972cdb7c08d1b1 0.11163700 BTC 9eb2833e8b30d12b28:0		
VIRTUAL SIZE	999 vKB	#8 688d298054e58c0142bc4f9dff33332ebb38b7101aad 0.10999832 BTC 37315d9199952c62e3cd:0		
WEIGHT UNITS	3993.575 KWU		2 CONFIRMATIO	NS 1.03230000 BTC



Case Study: SHA-256

Note: infeasible to use a random function in practice.

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Code for SHA-256

Pre-processing (Padding):	Initialize hash values:	Comprogram function main loop.
begin with the original message of length L bits	(first 32 hits of the fractional narts of the square roots of the first 8 primes 2 191)	for i far 0 to C2
append a single '1' bit	(1116 SE SICE OF CHE TRECIONAL PARES OF CHE SQUARE FOUSS OF CHE TISE & PINES 2.115).	for 1 from 0 to 63
append K '0' bits, where K is the minimum number \geq 0 such that L + 1 + K + 64 is a	10 :- 0x0a03e007	SI := (e rightrotate b) xor (e rightrotate 11) xor (e rightrotate 25)
multiple of 512		ch := (e and f) xor ((not e) and g)
append L as a 64-bit big-endian integer, making the total post-processed length a multip	h2 := 0x3c6ef372	temp1 := h + S1 + ch + k[i] + w[i]
of 512 bits	h3 := 0xa54ff53a	S0 := (a rightrotate 2) xor (a rightrotate 13) xor (a rightrotate 22)
	h4 := 0x510e527f	maj := (a and b) xor (a and c) xor (b and c)
Process the message in successive 512-bit chunks:	h5 := 0x9b05688c	temp2 := S0 + maj
break message into 512-bit chunks	h6 := 0x1f83d9ab	
for each chunk	h7 := 0x5be0cd19	h := q
create a 64-entry message schedule array w[063] of 32-bit words		a := f
(The initial values in w[063] don't matter, so many implementations zero them here	Initialize array of round constants:	f := e
copy chunk into first 16 words w[015] of the message schedule array	(first 32 bits of the fractional parts of the cube roots of the first 64 primes 2311):	e := d + templ
	k(063) :=	d := c
Extend the first 16 words into the remaining 48 words w[1663] of the message schec	0v428a2f98, 0v71374491, 0v55c0fbcf, 0va9b5dba5, 0v3956c25b, 0v59f111f1, 0v923f82a4	
array:	Avabla5ad5	6 - D
for 1 from 16 to 63	VABDICJEUJ, Ovd0072500 Ov12025b01 Ov242105ba Ov55047da2 Ov72ba5d74 Ov00dab16a Ov0bda06a7	D :- a
s0 := (W[1-15] rightrotate 7) xor (W[1-15] rightrotate 18) xor (W[1-15] rightshi	0x060/dd36, 0x12655D01, 0x245165D0, 0x550C/dC5, 0x/2D05d/4, 0x60d0D110, 0x9DdC06d/,	a := temp1 + temp2
3)	UXCI9DII/4,	
s1 := (W[1-2] rightrotate 17) xor (W[1-2] rightrotate 19) xor (W[1-2] rightshift	0xe49b69c1, 0xefbe4786, 0x0fc19dc6, 0x240ca1cc, 0x2de92c6f, 0x4a7484aa, 0x5cb0a9dc,	Add the compressed chunk to the current hash value:
10) artik urusti 16) kusti 7) kusti 7)	Ox76f988da,	h0 := h0 + a
W[1] := W[1-16] + SU + W[1-/] + SI	0x983e5152, 0xa831c66d, 0xb00327c8, 0xbf597fc7, 0xc6e00bf3, 0xd5a79147, 0x06ca6351,	h1 := h1 + b
· ///·	Ox14292967,	h2 := h2 + c
initialize working variables to current hash value:	0x27b70a85, 0x2e1b2138, 0x4d2c6dfc, 0x53380d13, 0x650a7354, 0x766a0abb, 0x81c2c92e,	h3 := h3 + d
a := nu	0x92722c85,	h4 := h4 + e
D := 11	0xa2bfe8al, 0xa81a664b, 0xc24b8b70, 0xc76c51a3, 0xd192e819, 0xd6990624, 0xf40e3585,	h5 := h5 + f
c := nz	Ox106aa070,	h6 := h6 + g
	0x19a4c116, 0x1e376c08, 0x2748774c, 0x34b0bcb5, 0x391c0cb3, 0x4ed8aa4a, 0x5b9cca4f,	h7 := h7 + h
c ;- uv	0x682e6ff3.	
r := b6	NY748f82ee. NY78a5636f. NY84c87814. NY8cc70208. NY90befffa. Nya4506ceb. Nybef9a3f7	Produce the final hash value (big-endian):
y - 10 h -= h7	Nynfilites, valvaluut, valterilit, valerilit, valvelila, valvulet, valvuletjaji,	direct := hash := h0 annend h1 annend h2 annend h3 annend h4 annend h5 annend h6 annend h6
11 += 11/		- argone - man - ne append at append at append an append at append an append at

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Canonical example: SHA-256 (used in Bitcoin, Solana, etc.).

 based on the Merkle-Damgard approach of iteratively applying a "compression function" to chunks of the input

- [assuming target output length = 256 bits]
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Compression function: resembles a block cipher (like AES).



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- deterministic, often < 100 lines of code
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Example: length-extension attacks.

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- given x and h(x), can construct a y:=xz that extends x so that can compute h(y) from h(x) (rather than recomputing h(y) from scratch)

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- given x and h(x), can construct a y:=xz that extends x so that can compute h(y) from h(x) (rather than recomputing h(y) from scratch)
 - lethal for HMACs, not obviously relevant to blockchain protocols
 - reason why SHA-256 often applied twice in the Bitcoin protocol?

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Tl;dr: practitioners treat cryptographic hash functions like SHA-256 as random functions, even though they're not.

- typically can get away with it, though beware of important edge cases

Recall: in e.g. Tendermint, validators pass around entire chains.

Protocol D (≈ Tendermint)

- at time $4\Delta \cdot v$:
 - each validator i sends its current chain A_i to v's leader ℓ
- at time $4\Delta \cdot v + \Delta$:
 - let A = of the A_i's received, the most recently created one; let B := all not-yet-included (in A) valid txs ℓ knows about
 - ℓ sends proposal (A,B) to all other validators
- at time $4\Delta \cdot v + 2\Delta$:
 - if validator i receives a proposal (A,B) from ℓ with A = A_i or with A more recent than A_i by this time:
 - send "(A,B) is up-to-date" message to all validators
- at time $4\Delta \cdot v + 3\Delta$:
 - if validator i has heard > 2n/3 "up-to-date" msgs for (A,B) by this time (a read quorum):
 - package these messages into a quorum certificate (QC), Q
 - send "ack (A,B,Q)" message to all validators and reset A_i := (A,B,Q)
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 - additional metadata, required for block to be viewed as valid

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- leader proposes a block B, not a chain
- "up-to-date" and "ack" messages reference has h(B), not B
 - quorum certificates attest to a blockhash, not a block (or a chain)

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- e.g., the predecessor blockhash in the current leader's block proposal
- e.g., the blockhash referenced in "up-to-date" and "ack" messages

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Good news: no, would contradict collision-resistance of hash fn.

– collision-resistant → "second pre-image resistant"

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Worry: what if validator can't find source for block?

• need to ensure this never happens ("data availability")