Lecture #12: Data Availability

COMS 4995-001: The Science of Blockchains URL: https://timroughgarden.org/s25/

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#### Goals for Lecture #12

- 1. Data availability committees.
  - offload storage responsibilities from validators to third parties
- 2. Verifiable information dispersal (VID).
  - use Merkle trees and Reed-Solomon codes to get minimal overhead
- 3. Data availability sampling.
  - how can an end user be confident that data is available?
  - idea: repeatedly download small random chunks

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Data availability (DA) problem: how to be sure that data really is being stored as intended?

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Implicit assumption: honest validators: (i) store entire blockchain; and (ii) upload past blocks to others on request. 13

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- example: Arbitrum AnyTrust (e.g., n=12, f=10)

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Solution: add redundancy to the m<sub>i</sub>'s using an erasure code.

- in effect, every server will store a little bit of info about every m<sub>i</sub>



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- if later remember only (3,14) and (4,19), can still recover line 5x-1



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- if store n > d+1 evaluations of f, can recover f from any d+1 of them
- easy to compute f from evaluations, e.g. via Lagrange interpolation
- corresponds to a *Reed-Solomon code* with parameters n and k=d+1

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- compute Merkle tree with values  $f(1), f(2), \dots, f(n)$  at leaves, check root = r
  - if not, sender committed to polynomial w/degree > k-1 (fail) 48

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Intuition: suppose crash failures (of servers) only.

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- ideally, reconstruct chunks whenever they're detected as missing 56