# Lecture #21: Nakamoto Consensus: Difficulty Adjustment, Block Rewards, and Selfish Mining

COMS 4995-001:
The Science of Blockchains
URL: https://timroughgarden.org/s25/

Tim Roughgarden

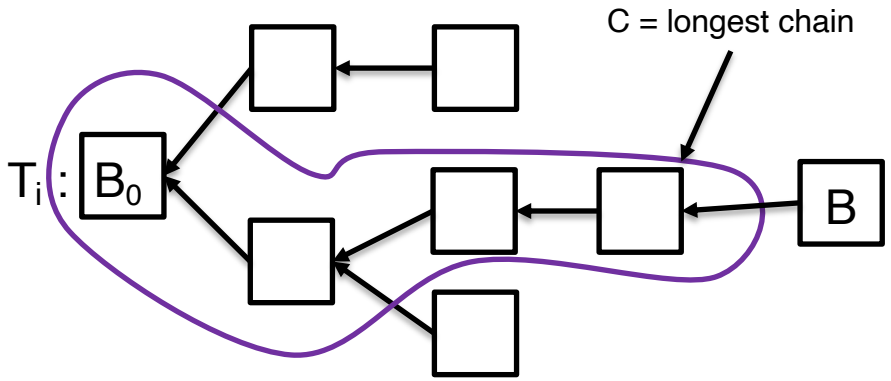
# Goals for Lecture #21

1. Difficulty adjustment in Nakamoto consensus.
   - tuning the difficulty threshold to achieve a target block rate
2. Limitations of proof-of-work.
   - drawbacks of Nakamoto consensus hold for PoW protocols generally
3. Why cryptocurrencies?
   - among other reasons, bootstrap a PoW blockchain protocol
4. Block rewards and selfish mining.
   - incentivizes block production; does it incentive other behavior even more?

# Recall: Nakamoto Consensus

Nakamoto consensus: longest-chain consensus + PoW leader selection.

- selection probability proportional to hashrate
- random length of a view = time for someone to solve puzzle



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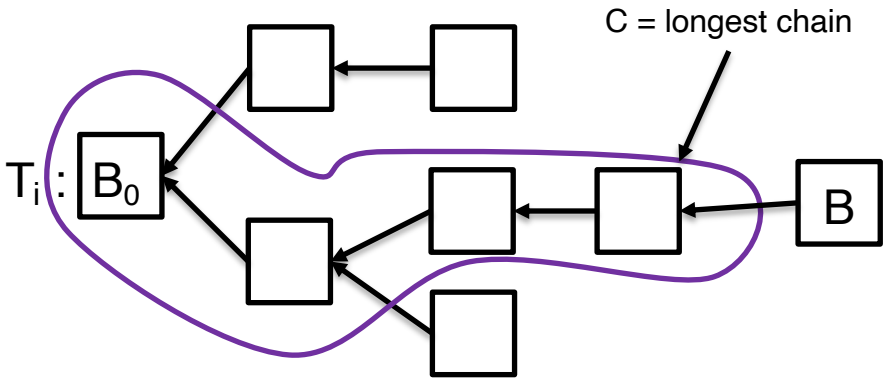


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Puzzle format: find x with h(x) ≤ t where:

- h = cryptographic hash fn, t = difficulty parameter (both protocol-defined)
- x = block header of the form < tx Merkle root II pred II pk II nonce >
  - point of the nonce: "grind" through possibilities until find a solution

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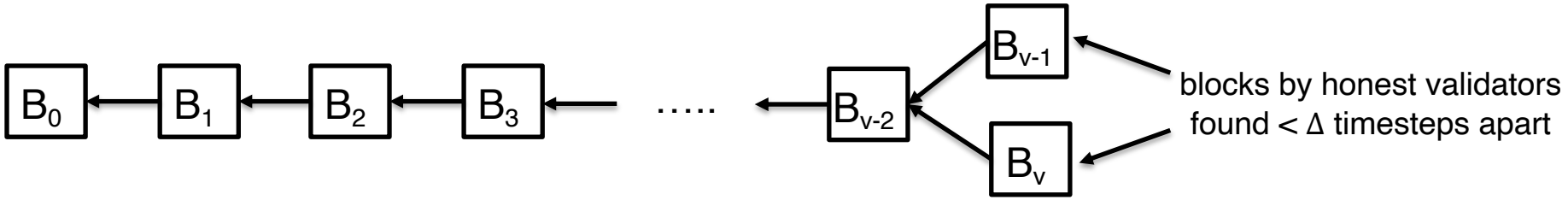

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Consequence: threshold most decrease (resp., increase) as total amount of hashrate increases (resp., decreases).

  - difficulty adjustment algorithm programmatically makes these updates

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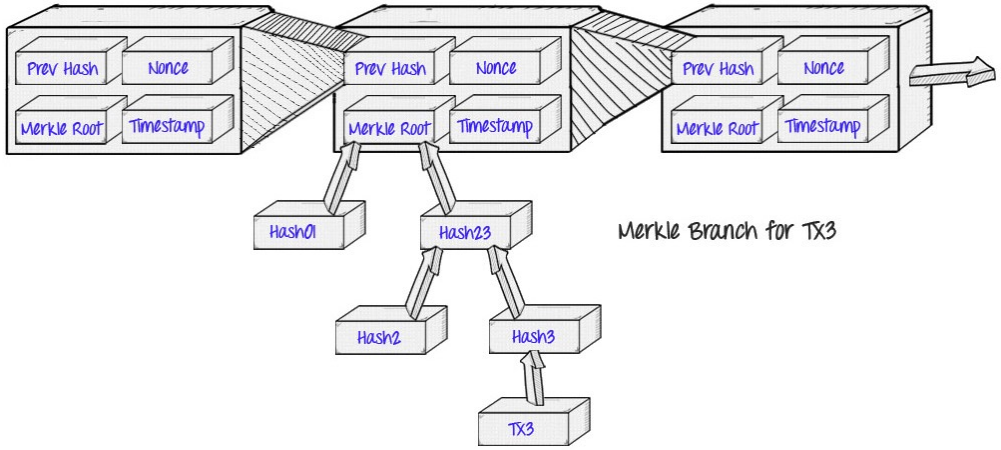

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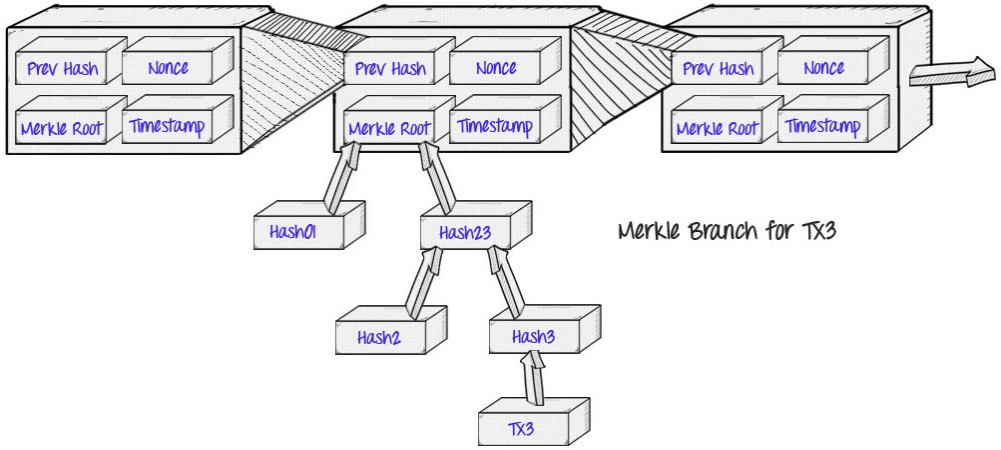
  - rules to limit timestamp manipulation by Byzantine validators (see HW7)



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  - if next epoch like previous one, expect to see 1 block/10 minutes
- time measured via timestamps in block headers

Also: need to redefine Nakamoto consensus so that (honest) leaders extend the *heaviest* chain (rather than the longest chain).

- weight of block with threshold t := $2^{256}$/t [expected # of attempts to obtain]
- weight of chain = sum of weights of blocks in the chain

# Recall: Guarantees for Nakamoto Consensus

Assumptions: (all necessary)

1. Synchronous network (i.e., max message delay ≤ Δ).
2. < 50% Byzantine *hashrate at all times*.
3. k large enough that, in every interval of ≥ 2k+2 views, a strict majority of the leaders are honest *with high probability*.
4. *difficulty threshold t small enough that avg view length >> Δ*

Guarantee: under these assumptions, Nakamoto consensus is consistent and live *with high probability*.

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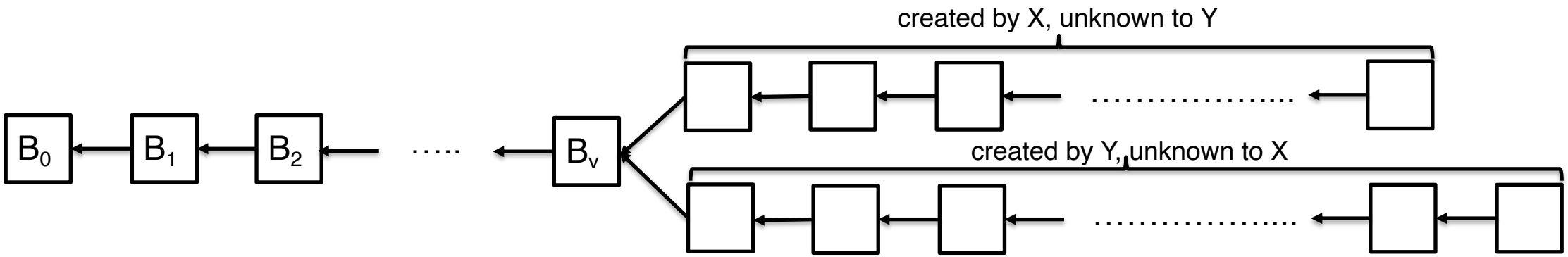

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Questions:

- tweak Nakamoto consensus so that one/both problems fixed?
- combine PoW with Tendermint rather than longest-chain?

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Upshot: drawbacks of Nakamoto consensus fundamental to all PoW protocols.

   - can be overcome (under extra assumptions) with proof-of-stake protocols

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Proof of (2): similar to proof of FLP Impossibility Theorem.

– churning validators can substitute for unbounded message delays

# Incentivizing Validators

Question: why run a validator (e.g., for a PoW protocol)?

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(ii) why a validator be “honest” (vs. profit-maximizing)?

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- hope: validators maximize rewards by following the protocol
- paid for getting blocks on the longest chain, not for being honest per se
- e.g., could validators have their blocks orphaned at different rates?

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- in general, yes! [Eyal/Sirer 14]

# A Profitable Deviation

Setup: adversary controls $\alpha < ½$ of the overall hashrate.

- other $1 - \alpha$ fraction obediently follows the protocol
- synchronous model with $\Delta$=0 (all msgs delivered instantly)
- assume ties broken in favor of adversary (can relax w/more work)

Profitable deviation from Nakamoto consensus:

- goal (i): get as many "A-blocks" on longest chain as possible (ideally, all of them)
- goal (ii): orphan as many "H-blocks" as possible, to maximize share of A-blocks on the longest chain

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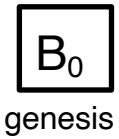

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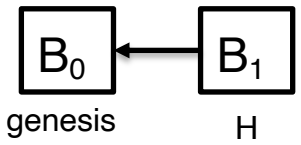

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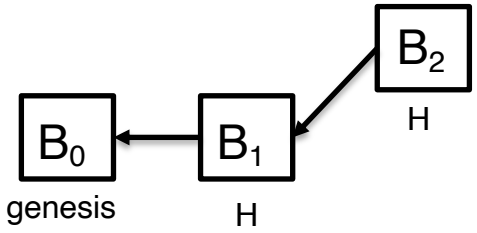

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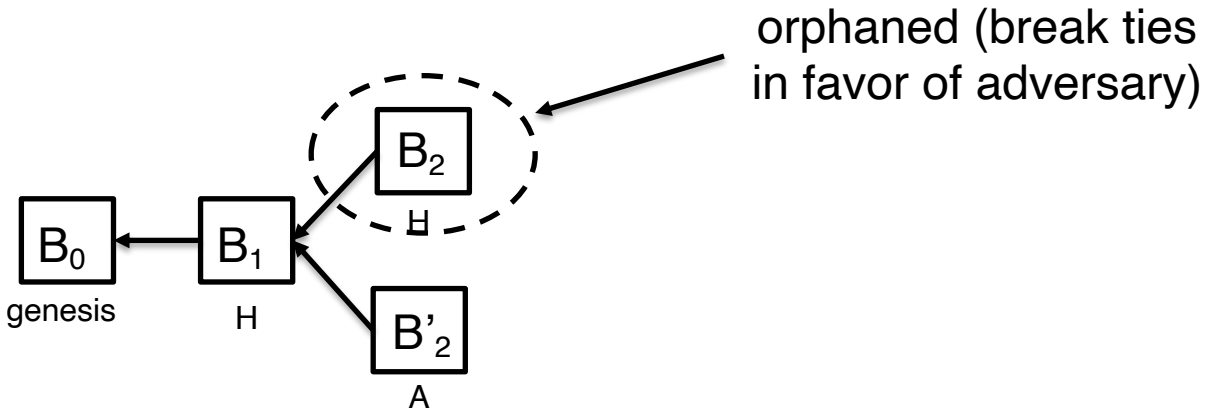

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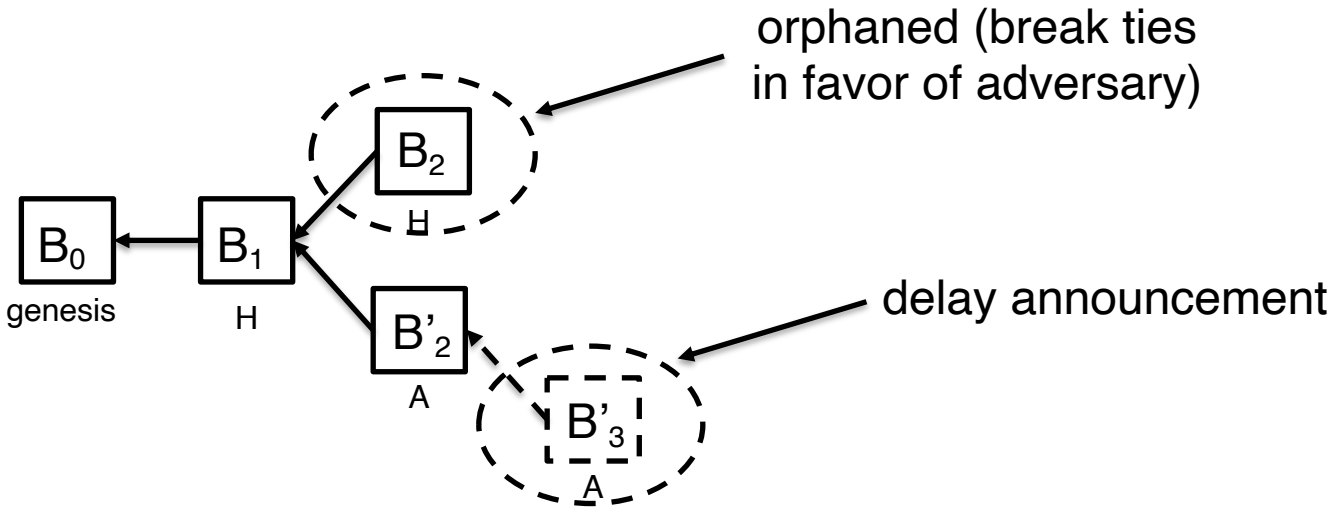

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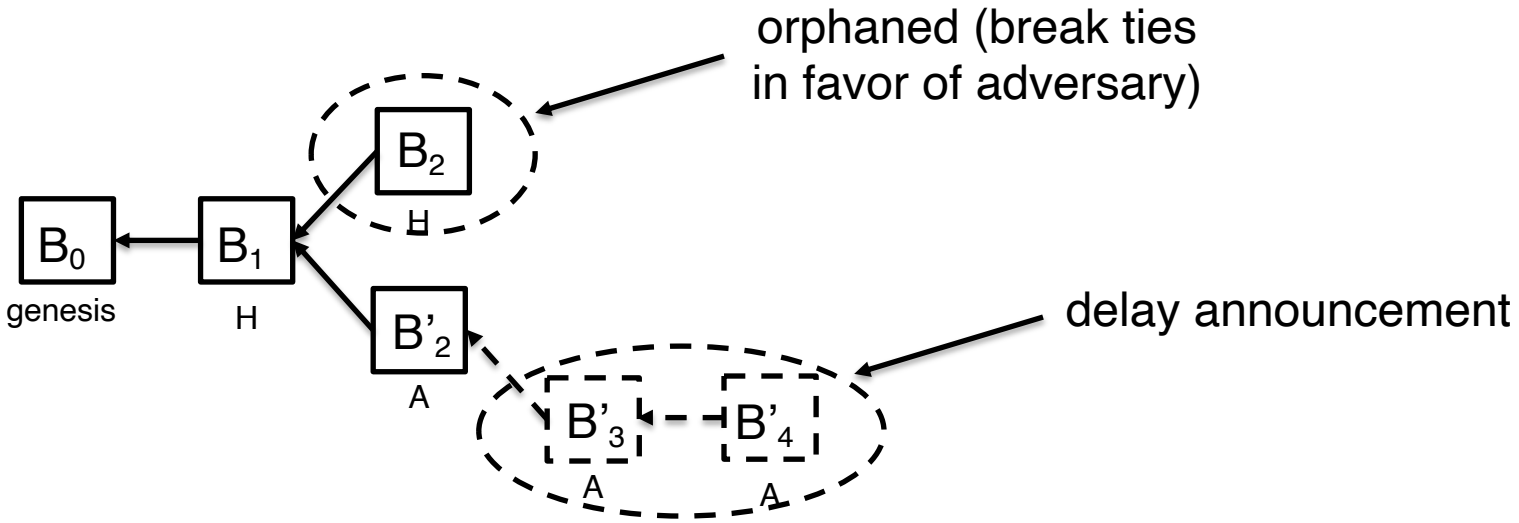

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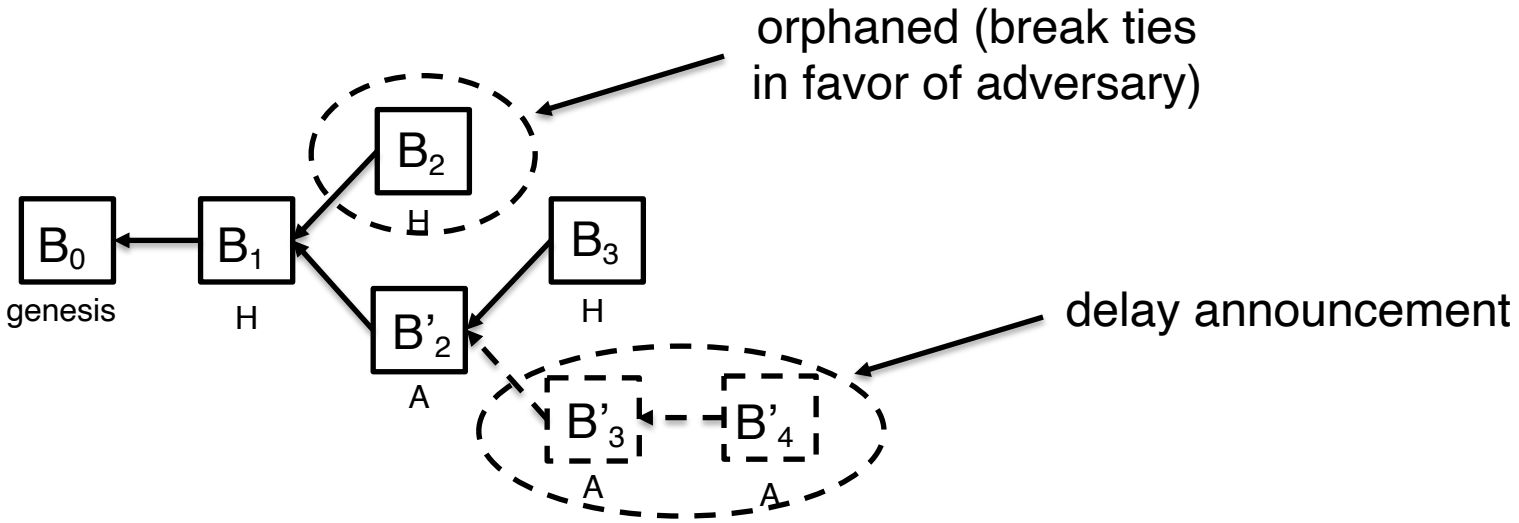

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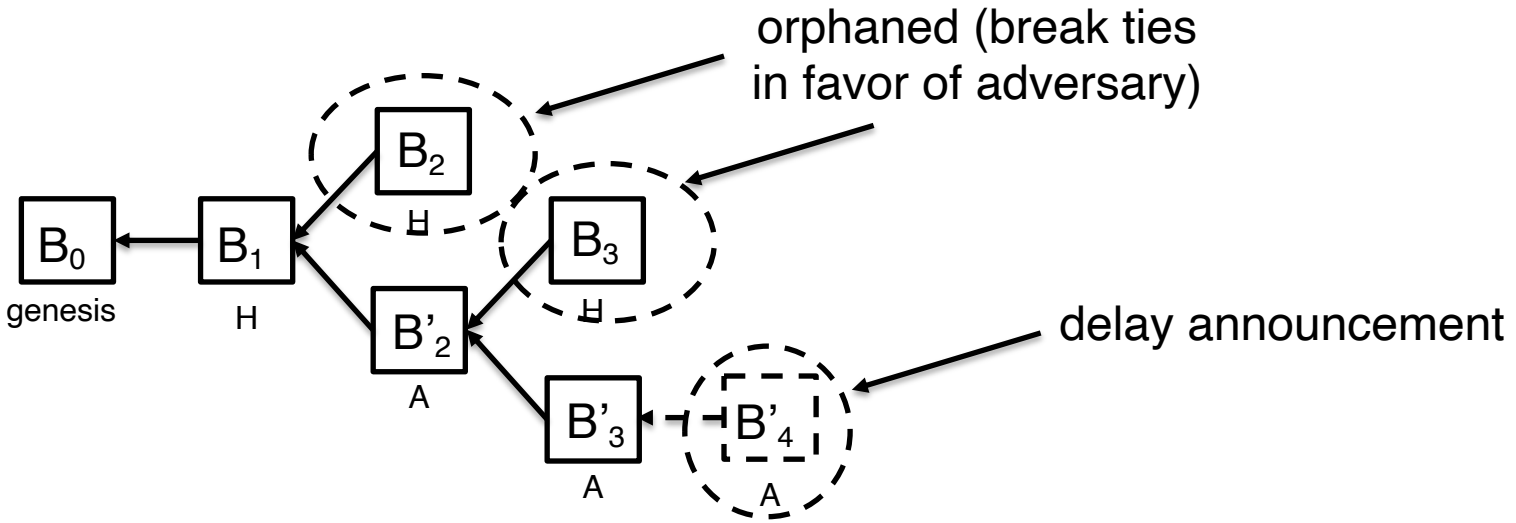

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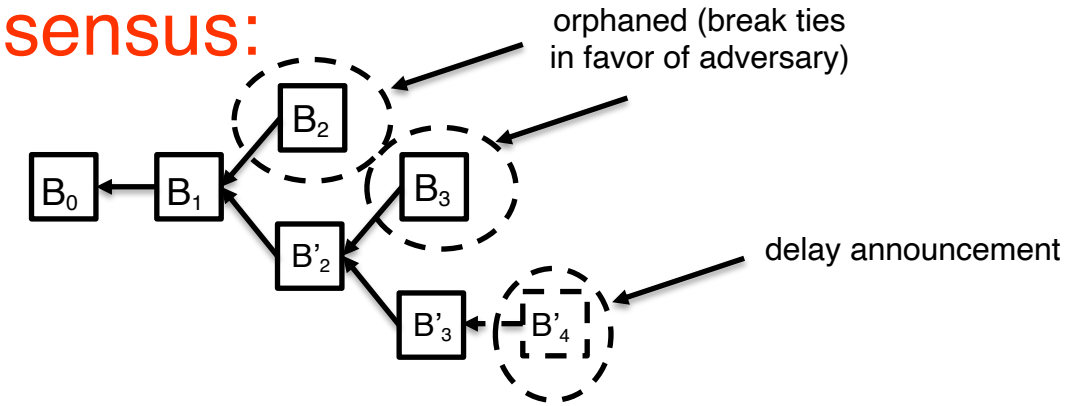

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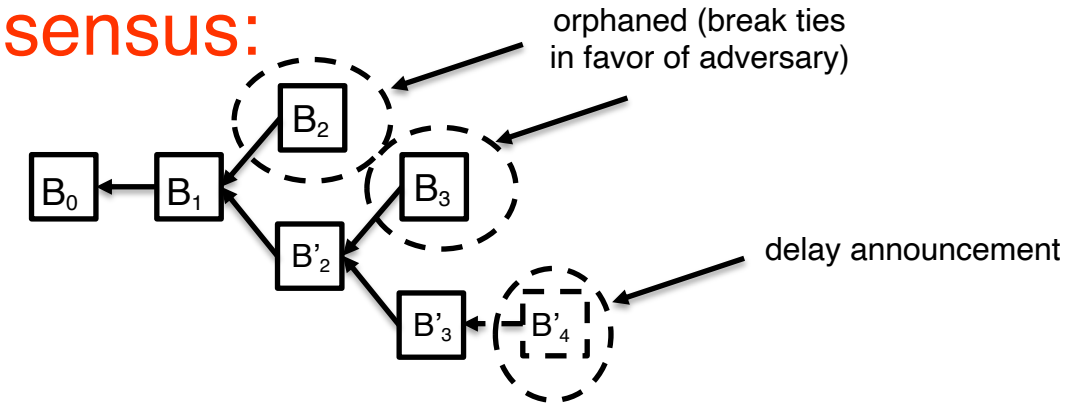

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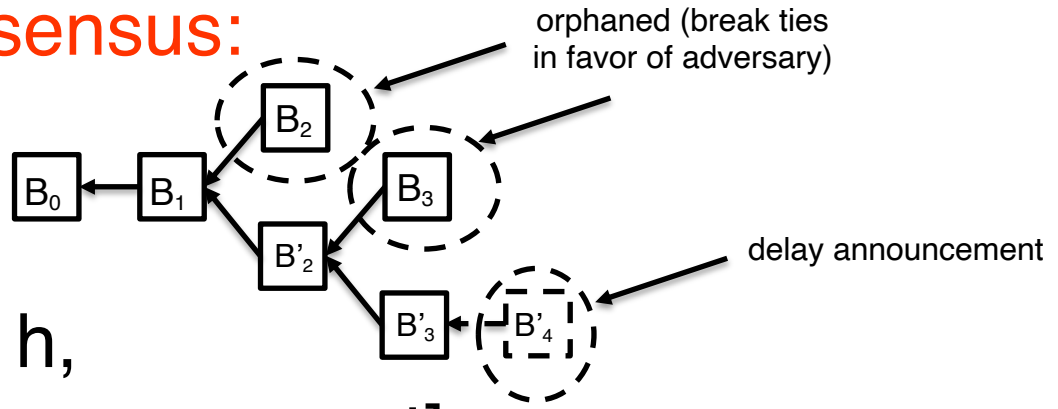

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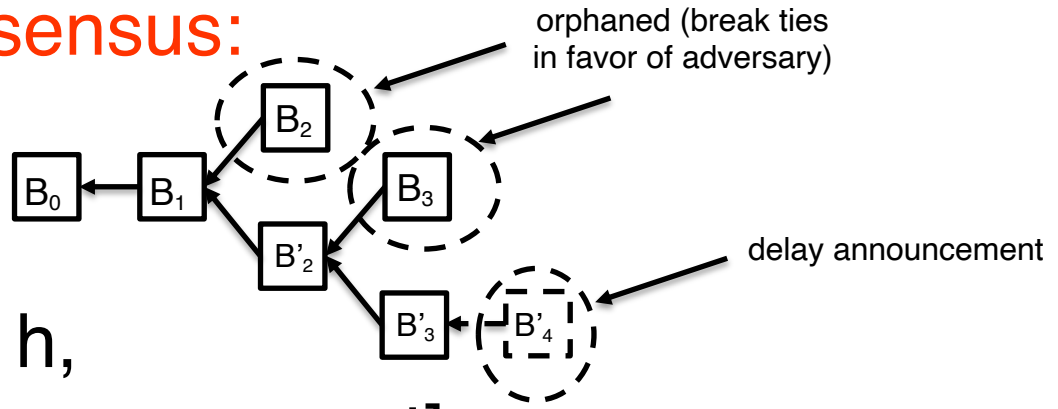

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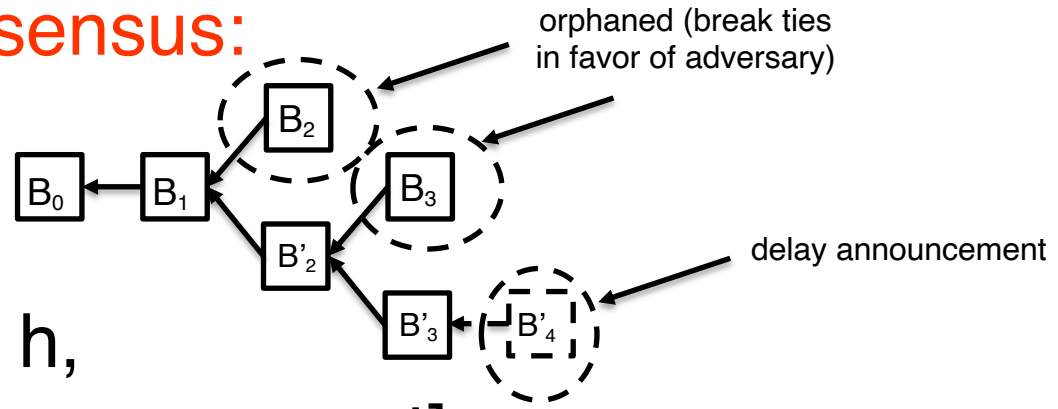


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- throughout: announce an A-block only once (+ immediately after) there in an H-block at the same height

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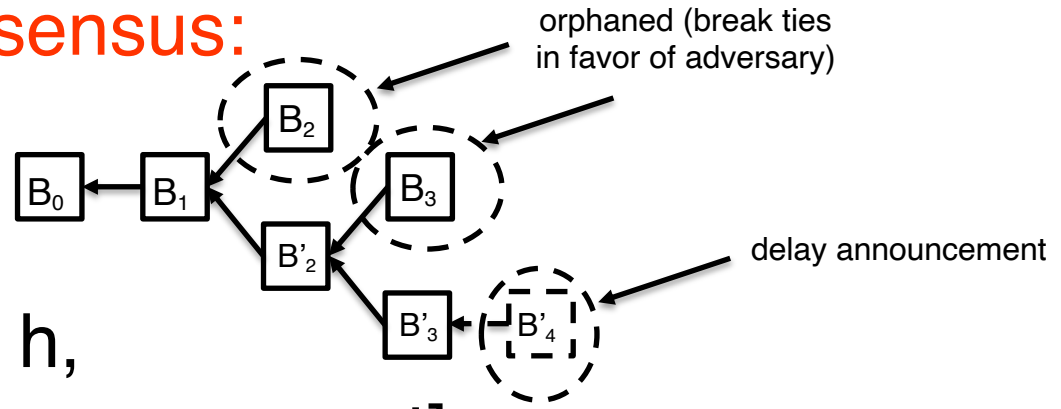
- case 1: if there is an A-block at height h, try to extend it [successful ➔ delay announcement]
- case 2: if only an H-block at height h, try to orphan it [successful ➔ announce immediately]
- throughout: announce an A-block only once (+ immediately after) there in an H-block at the same height
  - ➔ every A-block deployed to knock an H-block off of the longest chain



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Upshot: can boost rewards by deviating from intended behavior!