Lecture #7:
Longest-Chain Consensus

COMS 4995-001:
The Science of Blockchains

URL: https://timroughgarden.org/s25/

Tim Roughgarden

Two Categories of Blockchain Protocols

- "PBFT-type" protocols (e.g., Tendermint)
  - what we've been studying thus far
  - inspired by 20th-century consensus protocols (like PBFT [Castro/Liskov 99])

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  - one of several innovations in Bitcoin
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- perspective: design patterns with different consistency-liveness trade-offs; will fail in different ways (stalling vs. reorg/tx rollback)

Goals for Lecture #7

1. The essence of longest-chain consensus.
   - focus on permissioned implementation
   - will hint at proof-of-work permissionless version used in Bitcoin
2. Three drawbacks of longest-chain consensus.
   - loses consistency in asynchrony
   - even in synchrony, requires waiting to finalize transactions txs
   - Byzantine validators can control more than their fair share of blocks
3. Guarantees for longest-chain consensus.
   - consistent and live in synchrony with < 50% Byzantine validators

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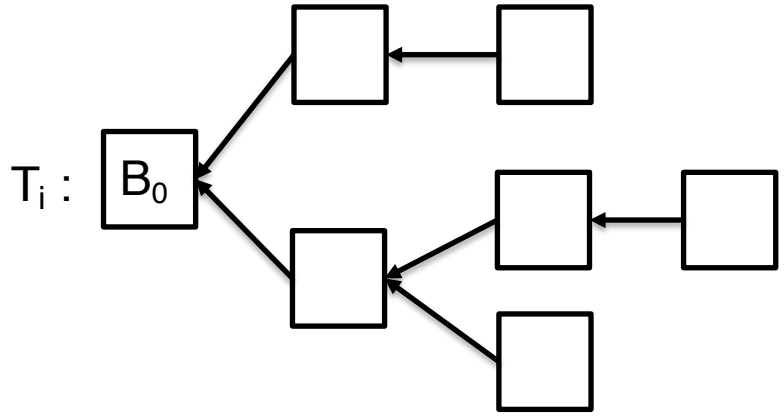

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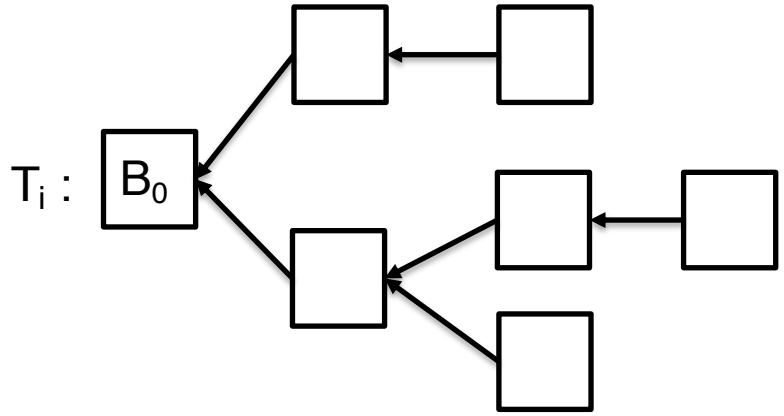

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  - block B is valid in view v if:
    - annotated with a view v' ≤ v
    - signed by leader of view v'
    - annotated with a predecessor block B'' from a view v'' < v'

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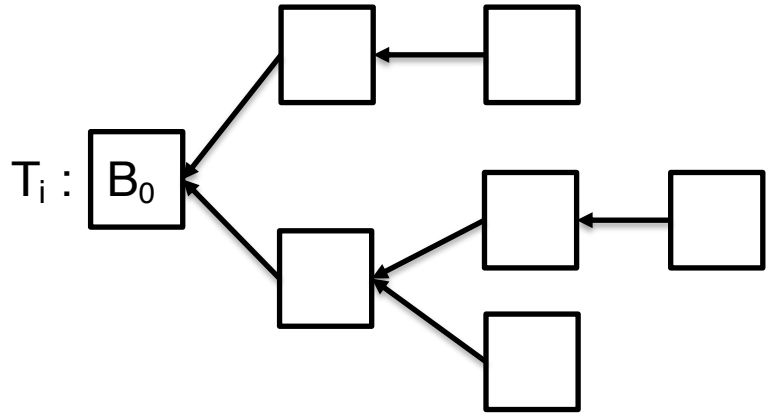

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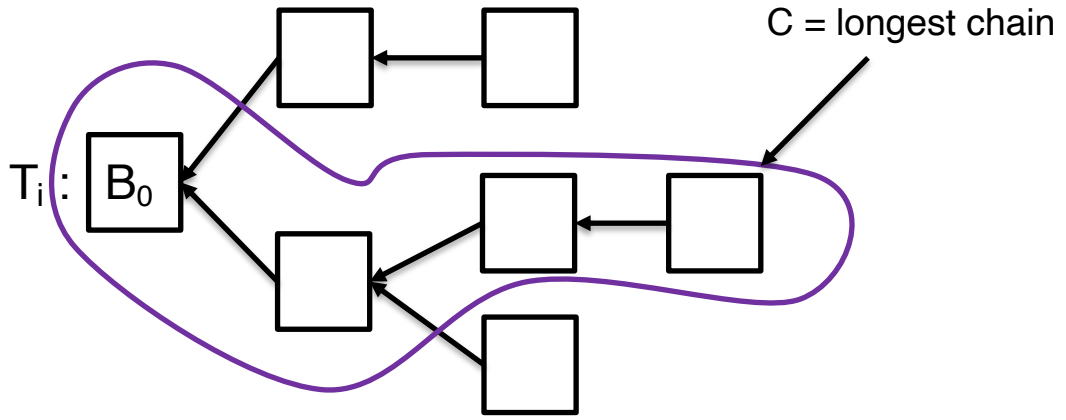

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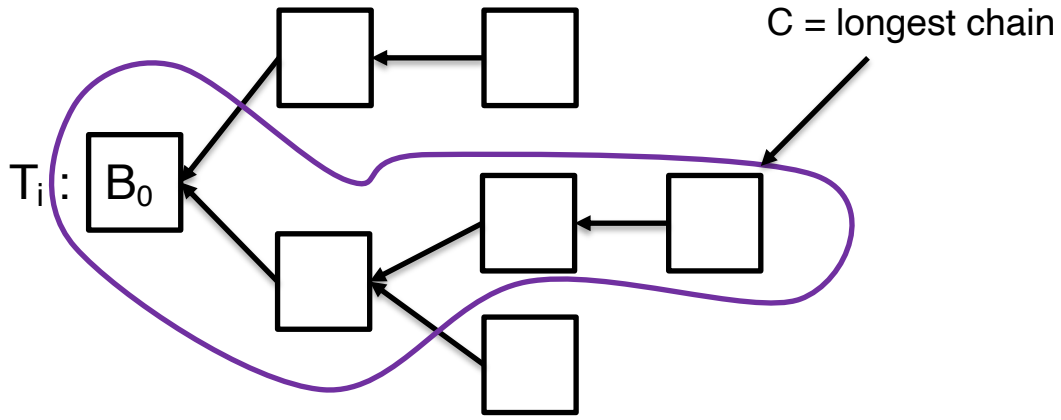

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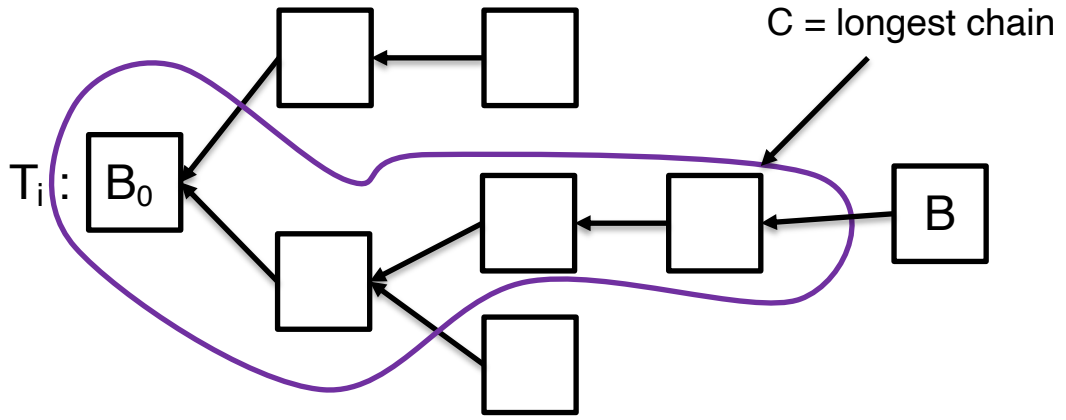

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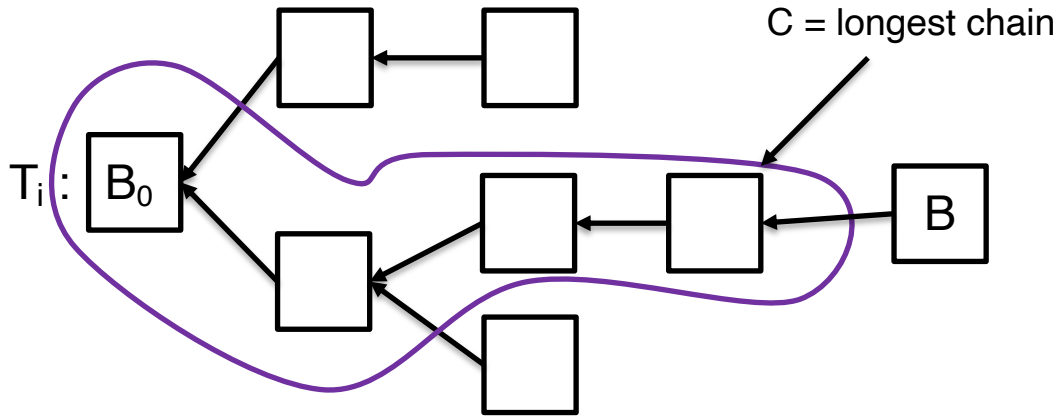

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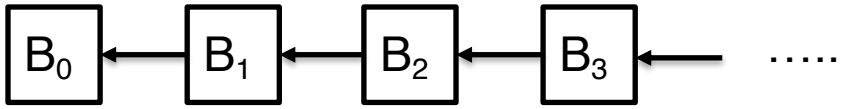

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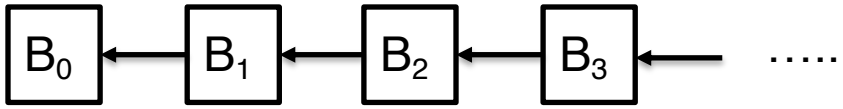

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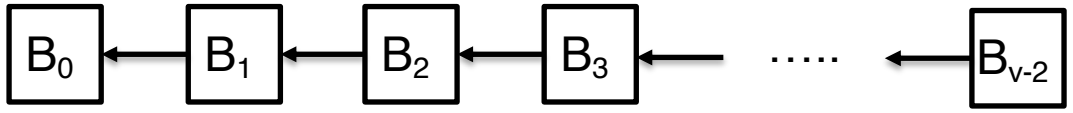

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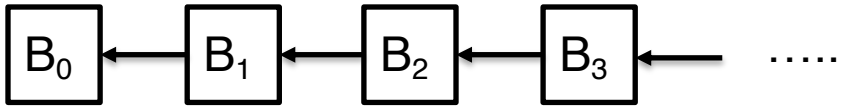

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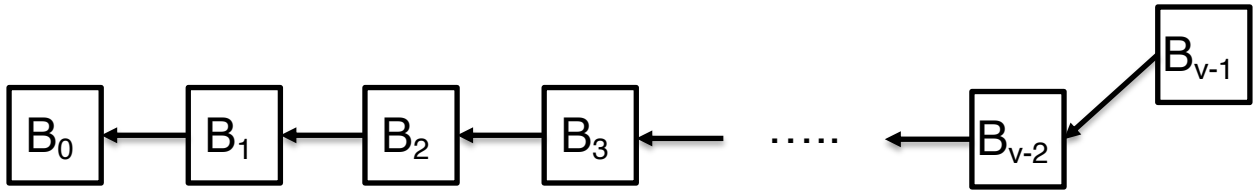

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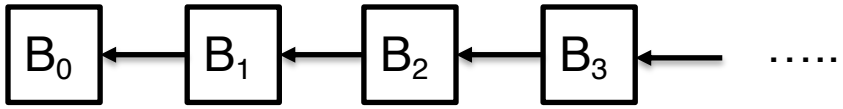

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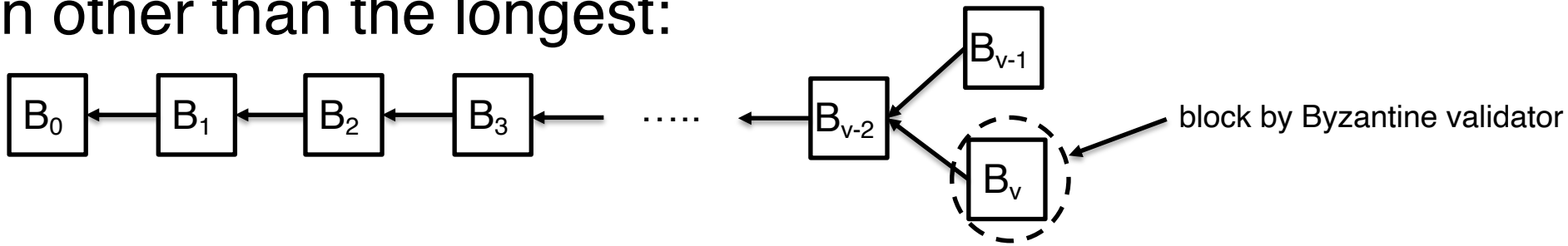

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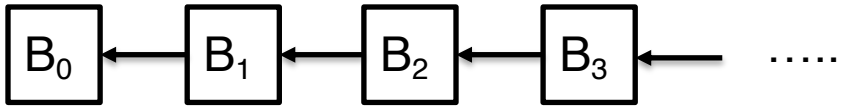

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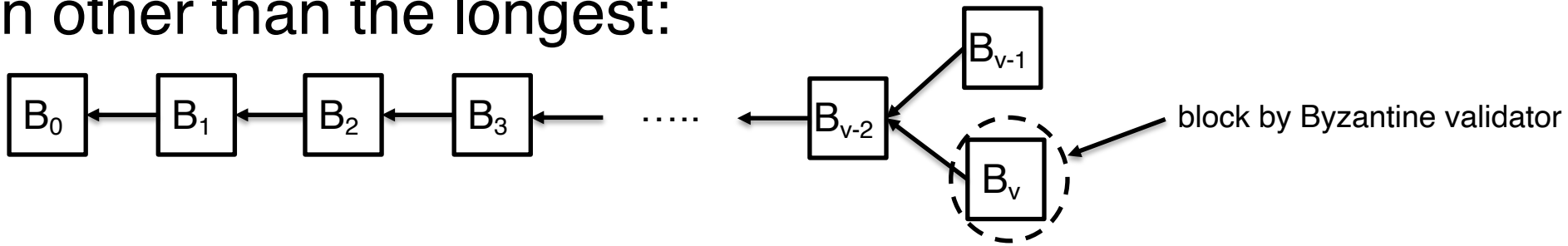

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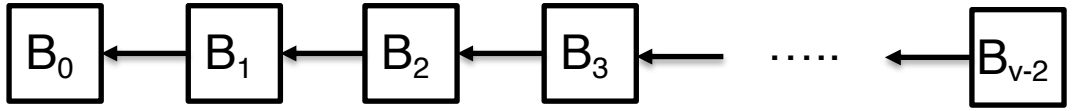

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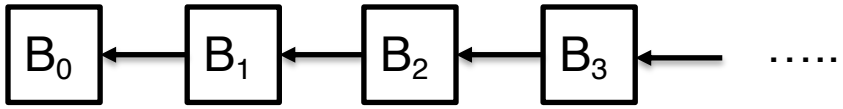

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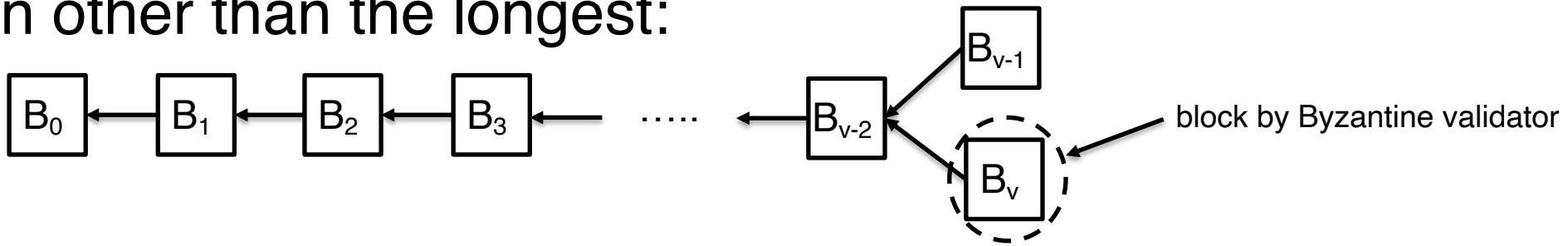

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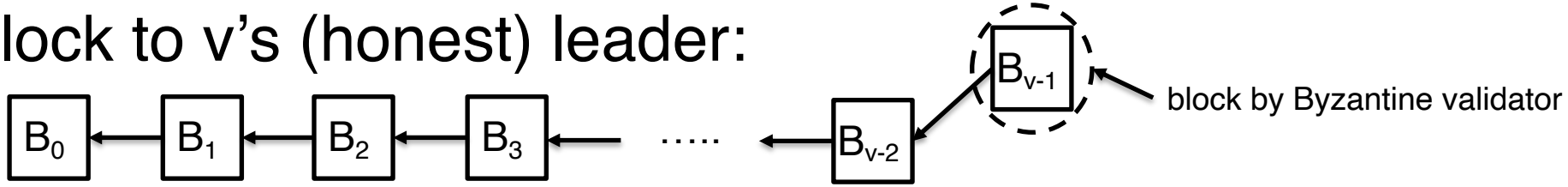

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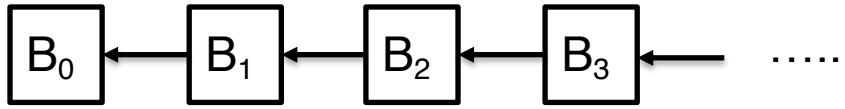

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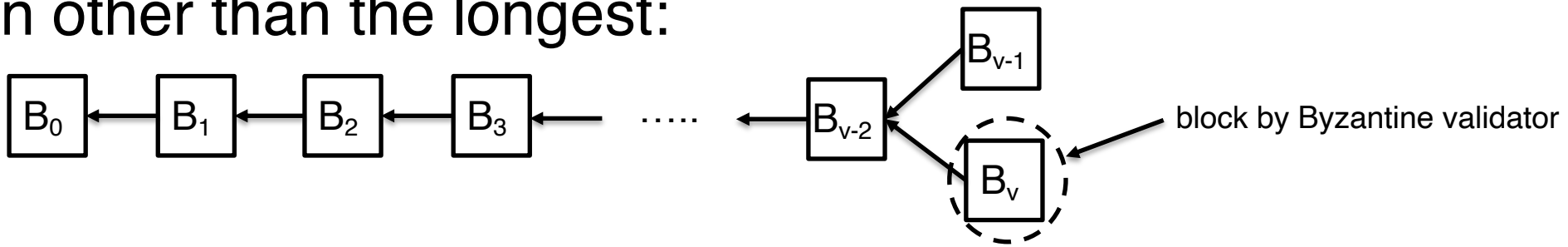

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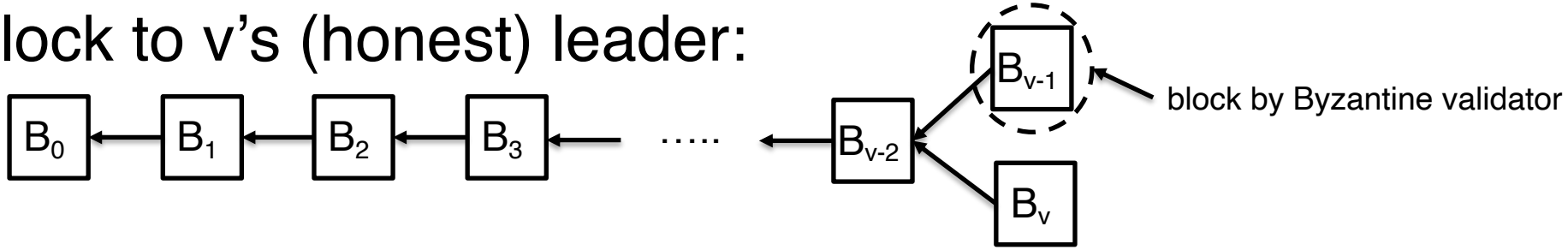

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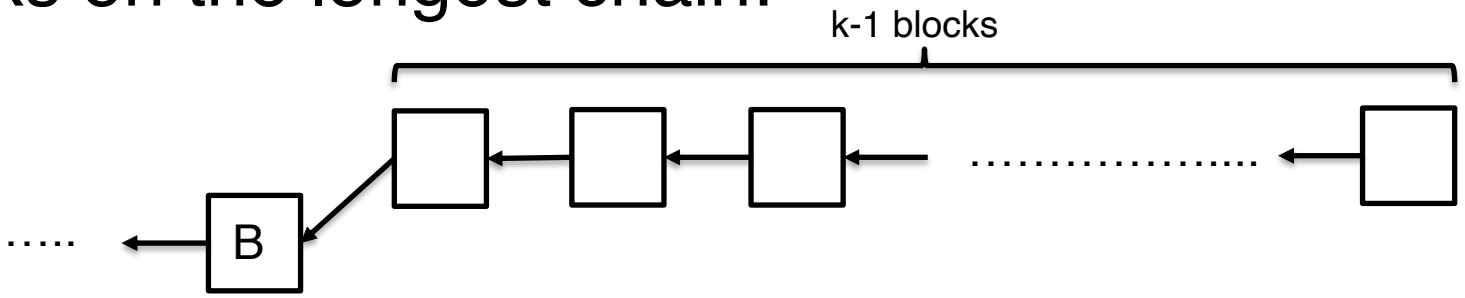

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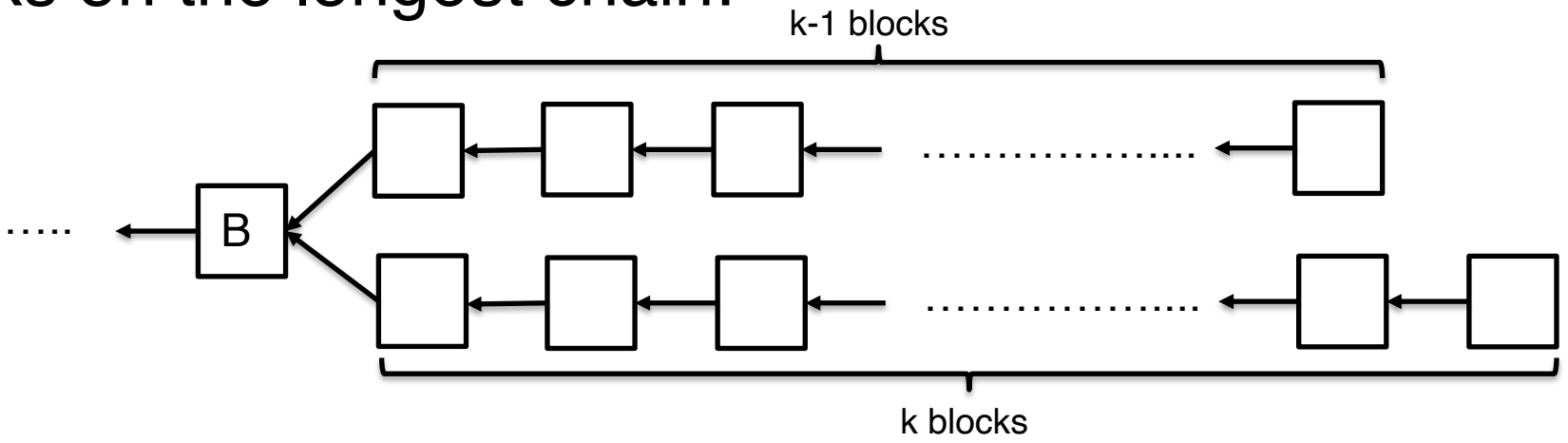

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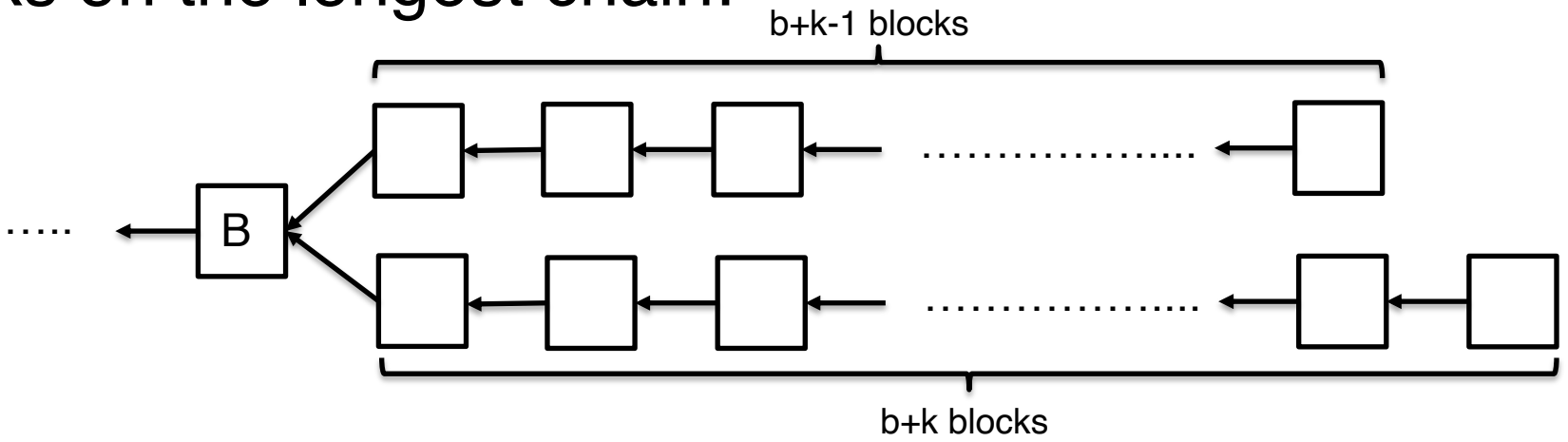

More generally: true for any interval in which Byzantine leaders outnumber honest leaders by ≥ k.

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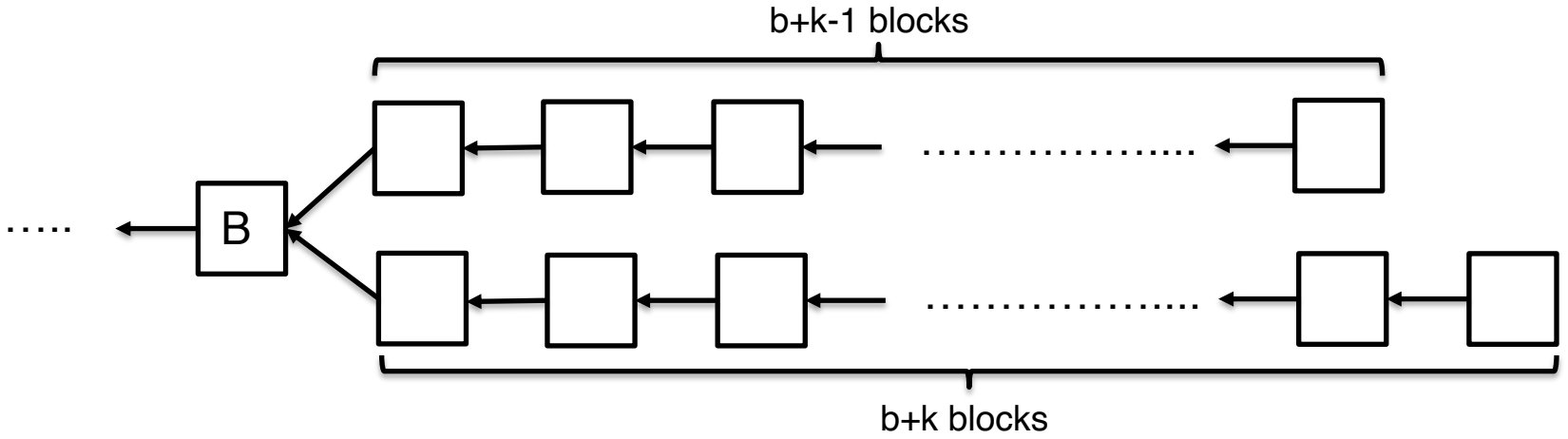

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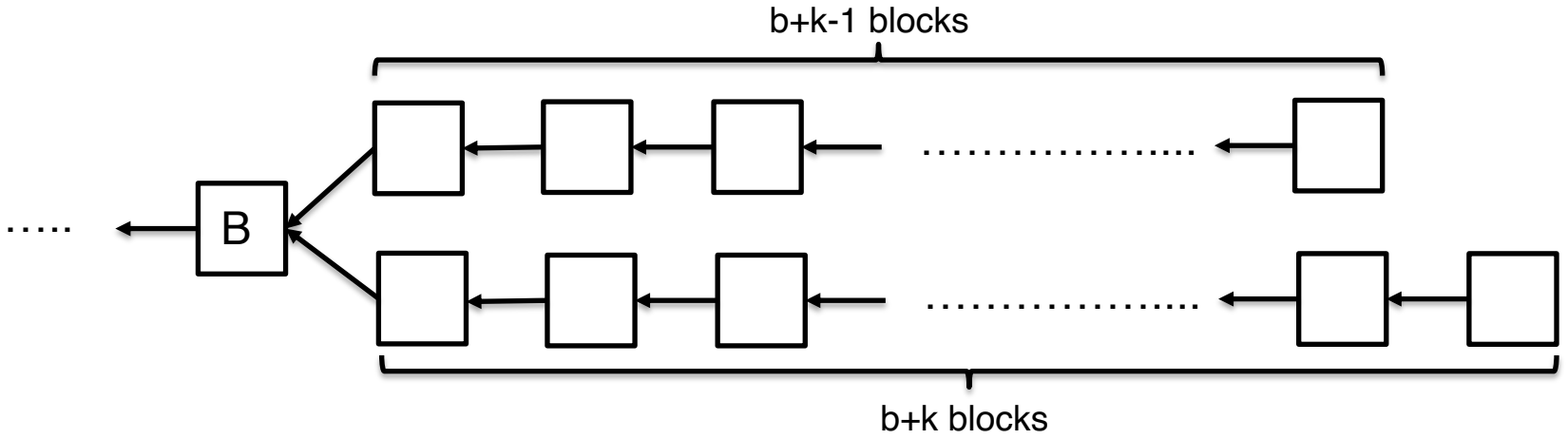

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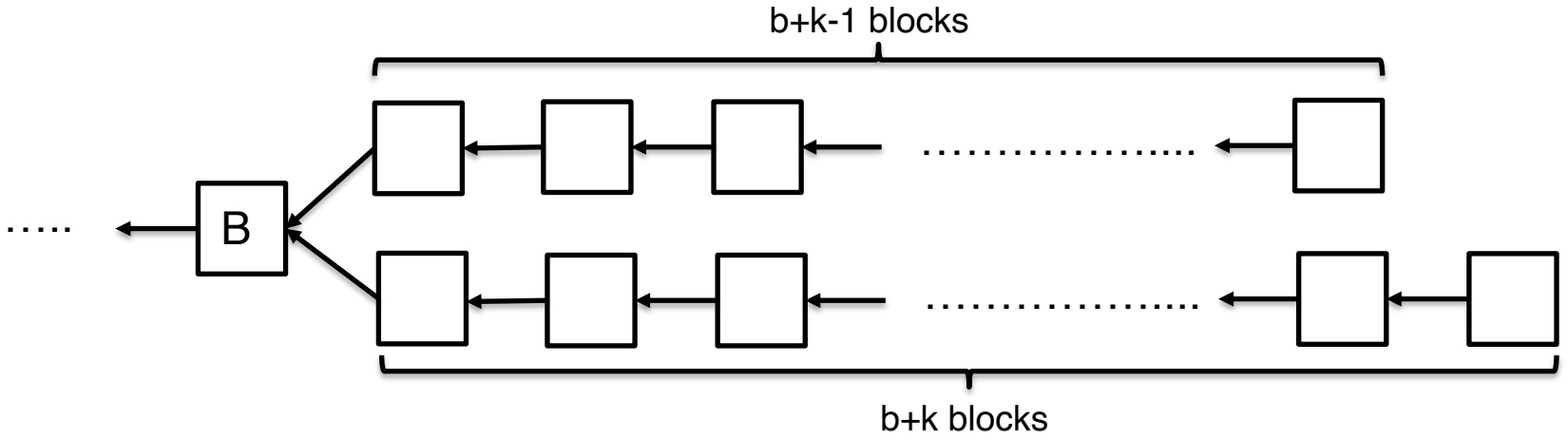

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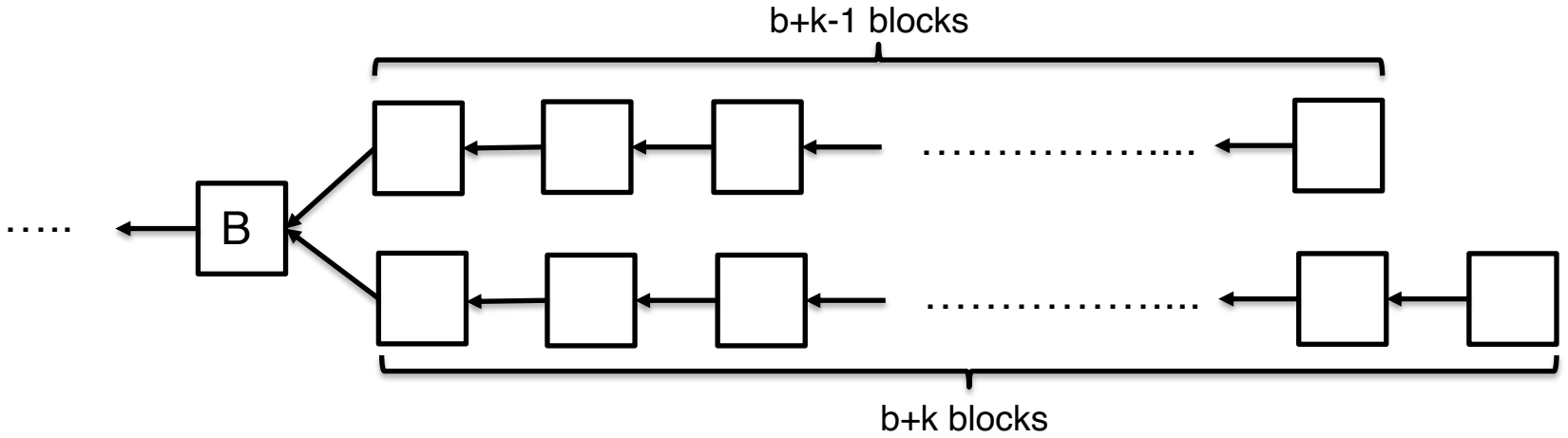

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  - ➔ always assume < 50% Byzantine validators in longest-chain consensus

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- folklore: for Bitcoin, k=6 (though Coinbase uses k=1)

Longest-Chain Consensus in Partial Synchrony

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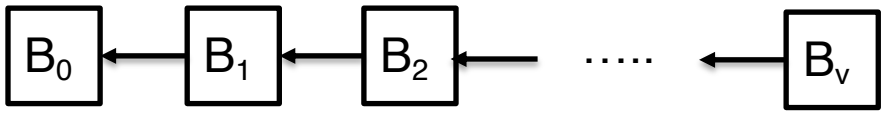

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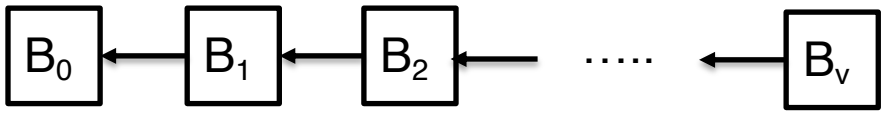

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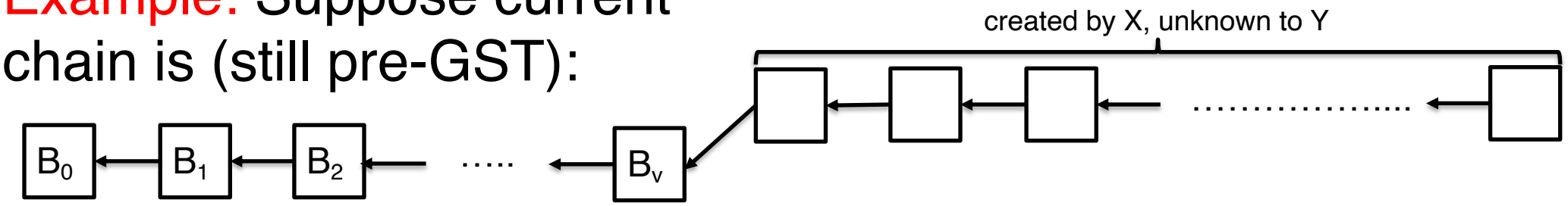

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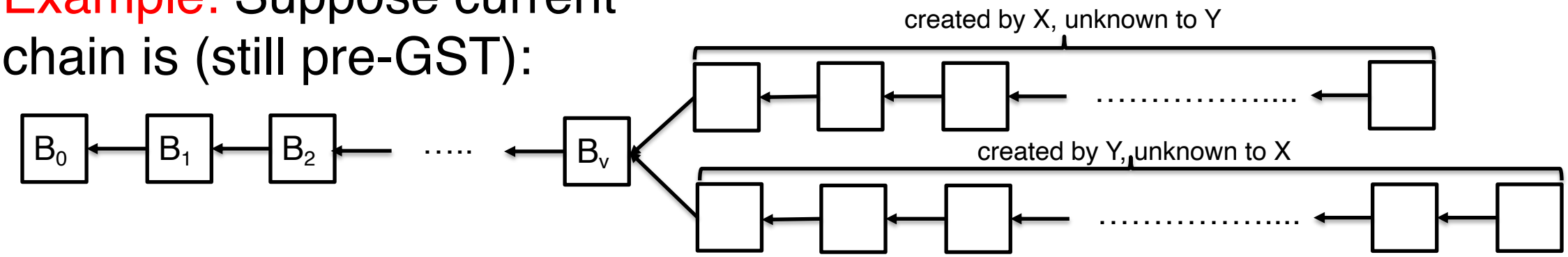

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  - Bitcoin: favors liveness despite hosting a valuable cryptocurrency (mismatch?)

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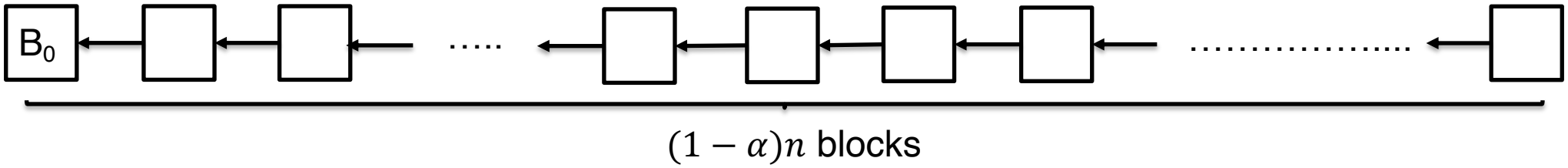

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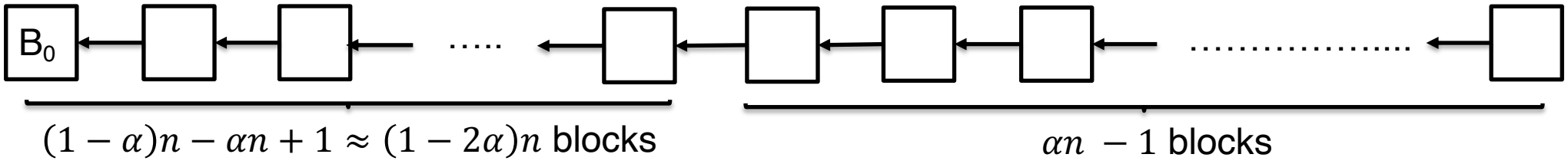

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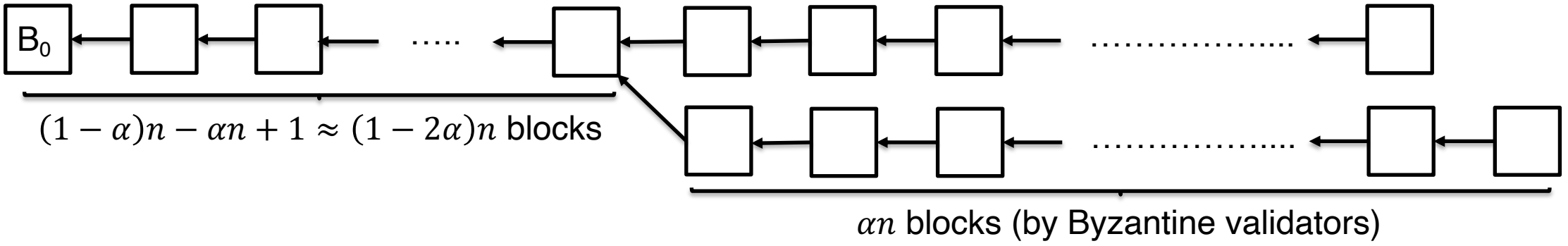

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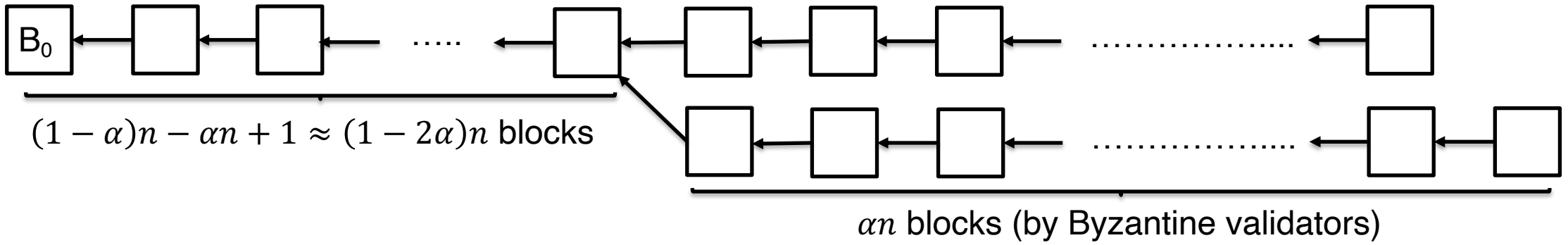

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Conclusion: longest-chain consensus is consistent, live, and guarantees chain quality $\geq \frac{1-2\alpha}{1-\alpha}$.

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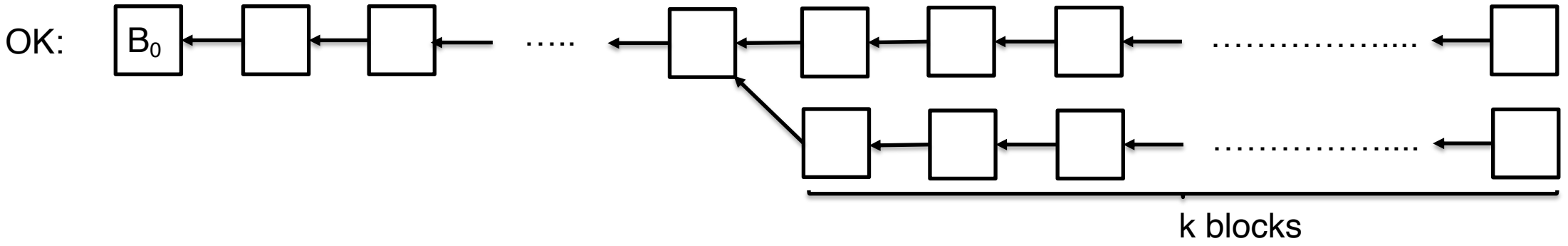

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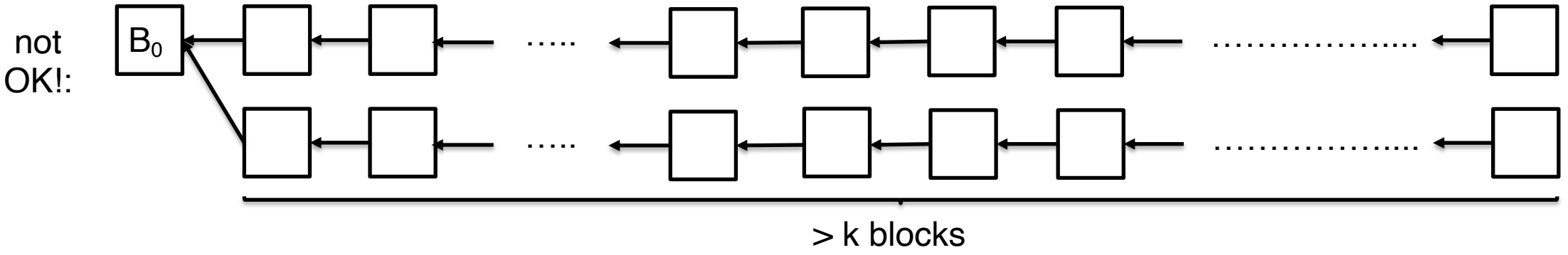

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Harder version: permissioned/proof-of-stake implementations.

- Byzantine leaders can equivocate, but guarantees still hold (harder proofs)