Proposer-builder separation (PBS) plays a major role in Ethereum's current block-building process. But is PBS a good design? Is it really necessary? A new paper with @bahrani_maryam @Tim_Roughgarden develops the theory necessary for reasoning rigorously about these questions

Centralization in Block Building and Proposer-Builder Separation

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Background: The idea of PBS is to split out the role of assembling a block of transactions from the other duties of the validator, with the goal of preserving decentralization in the validator set despite potential centralizing forces in block-building

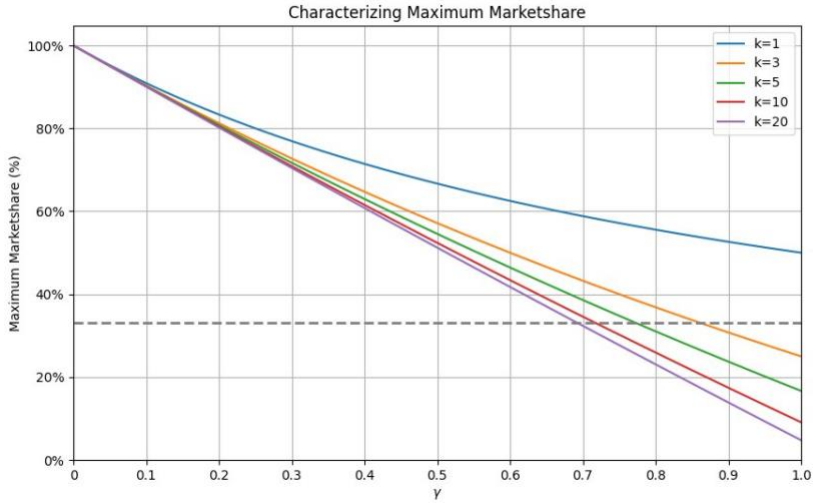
We begin with Ethereum circa 2020, in which each validator builds its own blocks. We're interested in the case in which some validators can extract more revenue from a block production opportunities than others

Possible reasons for such heterogeneity across validators include: private orderflow of transactions, better block-building algorithms, profit from off-chain events, etc

Is conventional wisdom on centralization forces in block-building correct? We probe this question mathematically in two different models

In model #1, we consider a game-theoretic model with endogenous staking, heterogeneous block producer rewards, and staking costs

Building on connections to Tullock contests, the main result here quantifies the concentration in the equilibrium staking distribution as a function of how many other validators k are within a fraction γ of their rewards



In model #2, we study a stochastic process in which heterogeneous block producers repeatedly reinvest rewards into staking

Building on connections to Pólya urns and Yule processes, the main result here quantifies, as a function of the block producer heterogeneity, how many blocks it takes before stake concentrates on the most sophisticated block producers

Theorem 4.2 (Bounds on Number of Blocks for ϵ-Centralization) *Let $\beta = \max\{\frac{\mu_1 r}{\pi_1}, \frac{\mu_2 r}{\pi_{-1}}\}$ and $\rho = \frac{\pi_{-1}}{\pi_1}$. Then for every $\epsilon > 0$:*

1. *(Upper bound on time to centralization) For every*

$$t > \frac{3}{2\mu_2 r}\left(\pi_1\left(\frac{3\rho(1-\epsilon)}{\epsilon}\right)^{\frac{\mu_1}{\mu_1-\mu_2}} + \pi_{-1}\left(\frac{3\rho(1-\epsilon)}{\epsilon}\right)^{\frac{\mu_2}{\mu_1-\mu_2}}\right),$$

we have

$$\Pr\left(x_{1,t} < 1-\epsilon\right) < 8\beta.$$

2. *(Lower bound on time to centralization) For every*

$$t < \frac{1}{2\mu_1 r}\left(\pi_1\left(\frac{\rho(1-\epsilon)}{3\epsilon}\right)^{\frac{\mu_1}{\mu_1-\mu_n}} + \pi_{-1}\left(\frac{\rho(1-\epsilon)}{3\epsilon}\right)^{\frac{\mu_n}{\mu_1-\mu_n}} - 2(\pi_1+\pi_{-1})\right),$$

we have

$$\Pr\left(x_{1,t} \geq 1-\epsilon\right) < 8\beta.$$

Thus economic and probabilistic arguments both show that leveling the playing field across validators can go a long way toward a more decentralized allocation of stake. But fast forwarding to present-day Ethereum, does PBS achieve such a leveling?

In our idealized model of PBS, k builders with i.i.d. block valuations compete in a first price auction. The proposer of a slot can also attempt to build its own block, with value drawn from a distribution dominated by the builder distribution

Our main result in this model of PBS is a positive one: under a common distributional assumption ("monotone hazard rate"), PBS ensures that no validator earns much more than another from a given block production opportunity

Theorem 5.1 (Competition Reduces Proposer Heterogeneity) *Let Y denote a set of k specialized builders, with rewards r_y drawn i.i.d. from a distribution D_Y that satisfies the MHR condition, and assume that builders bid according to the (unique) Bayes-Nash equilibrium of a symmetric first-price auction with value distribution D_Y. Let I denote a set of proposers, with proposer i's private block-building reward r_i drawn from a distribution D_i that is FOSD by D_Y. Assume that every proposer follows the strategy in (S1)–(S3). Then, for every pair $i, j \in I$ of proposers, the expected reward earned by i (conditioned on selection) is at most*

$$1 + O\left(\frac{1}{\log k}\right)$$

times that earned by j (conditioned on selection).

Tl;dr: our results formalize the forces toward centralization in block-building and the pushback provided by PBS. We hope that other researchers develop their own models and results to help the community better understand PBS and its pros and cons versus alternative designs.