CS269I: Incentives in Computer Science
Lecture #18: Prediction Markets*

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1 Markets for Information

1.1 Uncertain Events

Suppose we're interested in an uncertain event, such as whether or not the Warriors will win the 2017 NBA Championship, whether or not the winner of the next Presidential election will be a Republican or a Democrat, or whether or not 2017 will be the hottest year on record. Each of these events is uncertain, but eventually we will know whether or not they occurred. That is, the outcomes of these uncertain events are eventually verifiable. (Similar to the setup for scoring rules in the first half of last lecture, but different from the setup for eliciting subjective opinions in the second half.)

What's the best way to predict whether or not such events will happen? For example, suppose you had to predict the winner of a sporting match, despite knowing very little about the sport. You could do a lot worse than turning to the betting markets in Vegas for help. Vegas odds don't let you predict anything with anything close to full certainty, but even the best experts struggle to outperform betting markets with any consistency. For example, favorites cover the Vegas spread about 50% of the time, while underdogs beat the spread about 50% of the time.

1.2 The Iowa Electronic Markets

If betting markets work so well for forecasting the outcomes of sporting events, why not use them to also predict other kinds of uncertain events? This is exactly the idea behind the *Iowa Electronic Markets (IEM)*, which have operated since 1988 with the goal of using markets to forecast the outcome of presidential, congressional, and gubernatorial elections.

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The idea is simple. If the IEM wants to predict which of two candidates, A or B, will win a political election, it creates two fictitious securities (akin to stocks). Each share of the first security pays out \$1 if A wins the election, and 0 otherwise. Similarly, each share of the second security pays out \$1 if B wins the election, and 0 otherwise.

If your own personal opinion is that candidate A will win with probability p, then the obvious thing to do is to buy shares of the first security whenever its price is less than p (your expected value for them is p), and shares of the second security whenever its price is less than $1-p$ (your expected value for them is $1-p$). It also makes sense to sell shares that you own (at a profit) whenever the prices of the securities go above p and $1-p$, respectively.[1]

The idea is then to interpret the market price for the first (second) security as the market's "belief" about the probability that the first (second) candidate will win the election. For example, if the first security is trading at a price of \$.80 per share, then we can interpret the market as giving the first candidate an 80% chance of winning. Thus the market is effectively aggregating the beliefs of all of the participants and producing a "consensus" prediction.

Another useful type of security is one that pays out according to the percentage of votes that a candidate gets. For example, if candidate A receives 55% of the vote, then the per-share value of the corresponding security would be \$.55. Here, the market price of the security can be interpreted as the market's prediction for the vote share of a candidate.

Anyone can sign up with the IEM and start trading immediately. You might be wondering: isn't this basically online gambling (which is illegal)? The IEM circumvents regulation through a no-action letter granted by the Commodity Futures Trading Commission (CFTC), which effectively condones the IEM on the grounds that (i) it is non-profit and used solely for research purposes; and (ii) the stakes are small (a hard limit of \$500 per trader, with the number of traders typically in the low thousands).

1.3 Legality Issues

For a long time, the IEM were the only prediction markets that allowed U.S. residents to trade using real money. Several play-money prediction markets (where trades are in a fictitious currency) based on the IEM exist; since no real money changes hands, these face no regulatory issues. The Hollywood Stock Exchange, which predicts events like Oscar winners and box-office receipts, remains quite popular to this day.

There is no clear path to establishing legal real-money prediction markets, despite advocacy from a number of prominent economists (including Arrow, of Arrow's Impossibility Theorem [1]). InTrade was a real-money prediction market based in Ireland, which operated from 2001 to 2013. In 2012 the U.S. government prohibited U.S. residents from using InTrade, and the company shuttered its doors soon thereafter. Beyond the IEM, the only other real-money prediction markets that U.S. residents can currently legally trade in are run by the New Zealand-based company PredictIt. They received a no-action letter from the

[1] The prices of the two securities will always add up to a number very close to 1—otherwise there are very easy arbitrage opportunities that will be quickly exploited. For example, if the sum of the prices is less than 1, then no matter what you believe, you are guaranteed to make money by buying one share of each security (which then presumably raises the prices).

CFTC modeled after the one for the IEM. A trader can only spend up to $850 on PredictIt, and at most 5000 traders are allowed in each market.

1.4 Accuracy

Prediction markets are hardly the only way to predict uncertain events. For example, to predict the vote share of a political candidate, the tried-and-true method is to use polls—ask a bunch of random people who they plan to vote for, and then extrapolate to the whole population. Historically, prediction markets have often made more accurate predictions than polls, especially up to the last few days before an election (e.g., in terms of the difference between the predicted and actual vote share of a candidate) [7]. The paradox was that asking people what they thought *others* were going to do was more effective than asking them what they themselves would do. This is especially impressive given that the traders in the IEM are not a particularly representative group (for example, they are disproportionately Iowans).

2016, however, was not such a good year for prediction markets. The IEM gave Hillary Clinton an 80% chance of winning the Presidential election, similar to many polls, and higher than Five Thirty-Eight (which had it 65/35), which is effectively a smart averaging of different polls. Smart polling and poll aggregation techniques seem to be quite competitive with prediction markets, at least for major events like Presidential elections. It will be interesting to see if researchers can extract any lessons from the 2016 elections for building better prediction markets.

2 Continuous Double Auctions

So how do the Iowa Electronic Markets work, anyway? It turns out they use pretty much exactly the same trading mechanism as Wall Street (i.e., the New York Stock Exchange), called a *continuous double auction.* In a CDA, a trader can submit a buy or sell order at any time.[2] Such an order has three ingredients:[3]

1. A price. (For a buy order, the maximum acceptable sale price; for a sell order, the minimum acceptable sale price.)

2. The maximum number of shares to be bought/sold.[4]

3. An expiration date. (These play no role in this lecture.)

[2]How does anyone get any shares in the first place? In the IEM, a trader always has the option of spending $1 to purchase one share of every security corresponding to an event. So if you believe that the price of some security is too high, you can buy a package and then sell the corresponding share.

[3]Such orders correspond to limit orders on the NYSE. The IEM also supports market orders (where you buy or sell a specified quantity at whatever the going market price is), but these are not important for our story.

[4]In the IEM, you can only sell shares that you already own—i.e., short selling is not allowed. You can also only buy shares with money that you already have—i.e., buying on the margin is not allowed.

Trades are executed in a greedy fashion—whenever there is a buy order with price at least that of some sell order, a trade is executed. The various nuances of how this is done are best illustrated with an example. Suppose the following sequence of buy and sell orders shows up.

1. A buy order for up to 10 shares at a price of up to .5 per share.

   [No trades occur, since no seller is available.]

2. A buy order for up to 5 shares at a price of up to .6 per share.

   [Ditto.]

3. A sell order for up to 10 shares at a price of (at least) .7.

   [Since this selling price is larger than that of all of the outstanding buy orders, no trades occur.]

4. A sell order for up to 10 shares at a price of .5. The arrival of this sell order triggers a sequence of events:

   (a) There are two outstanding buy orders with price at least that of this new sell order. A trade should occur, but who gets priority? Answer: the buy order with the higher price (i.e., order #2). Thus 5 shares are transferred from the seller in order #4 to the buyer in order #2.

   (b) Since the buyer is willing to pay up to .6 and the seller is willing to sell down to .5, there is ambiguity about what the price should be. In the IEM, the price from the older order is used. Here, this means that the 5 shares will be sold at a price of .6, to the benefit of the seller.[5]

   (c) Since the number of shares desired in order #2 (5) is less than that on offer in order #4 (10), the sell order is only partially executed. The reduced sell order (up to 5 shares at price .5) is automatically resubmitted to the market, replacing the original order.

   (d) The remaining 5 shares from sell order #4 are now sold to the buyer in buy order #1, at a price of .5 per share. The buy order only gets partially executed, and the reduced buy order (up to 5 shares at a price up to .5) is automatically resubmitted to the market, replacing the original order.

5. Finally, suppose a new buy order comes in, for up to 5 shares at a price up to .8. This order is matched with the sell order #3, and 5 shares get sold at a price of .7 (the older of the two prices .7 and .8, again favoring the newer order). The remaining 5 shares of sell order #3 remain on the market.

[5] This is not how it works on the NYSE—there, the broker would collect .6 per share from the buyer and pay .5 per share to the seller (pocketing the difference, known as the "vig").

The incentives of a CDA are as discussed before: it makes sense to buy at any price less than your own estimate of the probability of the corresponding outcome, and to sell at any higher price.[6]

3 The Rise, Fall, and Rise of Prediction Markets

In the 20th century, the IEM was an oddity, known primarily to political nerds. Early this century, two events generated a lot of mainstream media coverage (bad in one case, good in the other) and significantly raised the profile of prediction markets.

3.1 The Policy Analysis Market

The first event was a scandal involving the Policy Analysis Market (PAM). This was a pilot program funded by DARPA (the research wing of the Defense Department), on using prediction markets to help with government and military decisions. The likely plan was to have two different prediction markets. The first would be a secret market, open only to a small group of individuals working at the various intelligence agencies (CIA, FBI, NSA, etc.). This market could potentially be used to predict highly classified events (like whether or not the U.S. could successfully pull off a coup in some other country). The goal of this market was to make it easier for the different intelligence agencies to share information, dodging the usual political and territorial hurdles. (The DARPA program was funded a bit before 9/11, but post-9/11 better intelligence sharing was obviously at the forefront of everyone's mind.) The second prediction market was to be open to anyone, likely for the prediction of events related to economic and political stability in the Middle East (e.g., will the current leader of some country still be in power at the end of the year?).

In mid-2003, two U.S. Senators called a press conference to publicly denounce PAM, calling it a repugnant mechanism for "betting on terrorism." The story quickly went viral, and almost all of the media coverage was negative (see [6]). DARPA defunded the program less than 24 hours after the press conference, and the responsible DARPA division director (John Poindexter, better known for his role in the Iran-Contra scandal of the 1980s) resigned within a week. The future of prediction markets did not look good.

3.2 The Wisdom of Crowds

The second event was the 2004 publication of *The Wisdom of Crowds*, a bestselling book by James Surowiecki [8]. This book strongly advocated for the wider use of prediction markets, especially internally at corporations. There was a precursor to this suggestion: Hewlett-Packard, in the 1990s, ran an internal prediction market (open only to HP employees) to predict future printer sales. The results were promising, with the markets outperforming the internal experts' predictions six out of eight times. More generally, one can imagine using

[6] When traders update their beliefs based on other traders' behavior, there can be opportunities for manipulation; see also Section 4.4.

internal prediction markets for all kinds of events relevant to a company, like whether or not some project will actually get shipped by the supposed deadline. Again, the primary goal is to enable the kind of information sharing across different groups that don't usually talk to each other very much. Coincidentally or not, in the mid-aughts a number of companies started using internal prediction markets (including Google, Microsoft, etc.), generally with play money, and there are currently companies that specialize in setting up such markets (e.g., ConsensusPoint and Cultivate Labs). Many of these internal prediction markets use a mechanism different from a continuous double auction (discussed next).

4 Market Scoring Rules

4.1 Motivation

In this section and the next we describe two equivalent mechanisms for running a prediction market that are different from continuous double auctions. Why not just always use CDAs? CDAs tend to work very well in "thick" markets, where there are lots of traders, but not in "thin" markets (with few traders) or "illiquid" markets (where there is a big "bid-ask spread," meaning a big gap between the highest outstanding buy order and the lowest outstanding sell order). For an extreme case, imagine that there is only one trader, who happens to have extremely accurate information. A CDA cannot elicit this information—with no other traders, there won't be any trades, and hence no "current market prices."

One approach to enabling trades in thin markets is to use an "automated market-maker." For us, this means that at all times the market posts some price, and is always happy to buy or sell shares at the current price. (Presumably when traders buy shares the price goes up, and when they sell shares it goes down.) Then, even with only one trader, that trader can report all of her information, by buying and selling shares to move the market prices to match her beliefs.

4.2 Definition

So how does one implement a prediction market with an automated market-maker? The key idea is to make use of the *strictly proper scoring rules* introduced in the last lecture. After all, our assumption is that the event being predicted is eventually verifiable (as is needed to use a scoring rule), and we know that such rules can elicit truthful predictions when there is a single participant (which is the bad case for CDAs).

A scoring rule is designed for a single player, whereas prediction markets can have many. How can we extend scoring rules to the multi-player case? One idea is just to use a separate scoring rule for each player. But this has two problems: (i) if you receive conflicting beliefs from different players, you're stuck figuring out how to aggregate them into a single prediction (which was supposed to be the job of the prediction market); and (ii) a separate payout must be made to each player, in some quantity that players care about.

The way to address both of these issues is to use a single scoring rule that is shared by all of the players. The idea leads to the notion of a *market scoring rule*.

Market Scoring Rule

- Let S be some strictly proper scoring rule.
- Initialize $\mathbf{p}^0$ to some probability distribution over the outcome set X. (E.g., the uniform distribution.)
- At each time step $t = 1, 2, \ldots$:
  - Any player can update $\mathbf{p}^{t-1}$ to an arbitrary probability distribution $\mathbf{p}^t$.
- After the outcome $i \in X$ is realized:
  - The payout to the player who made the tth update is

$$S(\mathbf{p}^t, i) - S(\mathbf{p}^{t-1}, i). \tag{1}$$

Thus a player is rewarded according to the extent that her report improved the current prediction (given that i was the outcome that occurred). If the player's report was less accurate than the previous one, then this reward will be negative, and the player will have to pay the market-maker. A player can guarantee herself a reward of 0 by not changing the current beliefs (i.e., setting $\mathbf{p}^t = \mathbf{p}^{t-1}$).

It may seem weird that the final prediction of a market scoring rule is just the last distribution that anyone reported. But this is analogous to how, in CDAs, the market price of a security depends only on the selling price of the most recent transaction involving that security. It's possible for the probability distribution of a prediction market to swing wildly with each new report, but in practice it tends to settle down (as the beliefs of different players converge, as players acquire better information, or just because of risk-aversion and budget constraints). The intuition for this may be clearer after we reinterpret market scoring rules as trading markets in Section 5.

4.3 Bounded Financial Loss

We next note two simple but important properties of market scoring rules.

Proposition 4.1 *The total payout to players with a market scoring rule is* $S(\mathbf{p}^T, i) - S(\mathbf{p}^0, i)$.

Proof: By definition, the total payout is $\sum_{t=1}^{T} (S(\mathbf{p}^t, i) - S(\mathbf{p}^{t-1}, i))$, which is a telescoping sum that evaluates to $S(\mathbf{p}^T, i) - S(\mathbf{p}^0, i)$. ■

For example, if S is the logarithmic scoring rule (with $S(\mathbf{p}, i) = \ln p_i$) and $\mathbf{p}^0$ is the uniform distribution, then the total payout of the market scoring rule will be at most $\ln |X|$ (as you should check).

So, an automated market-marker might lose money. (This is different from a CDA, which is guaranteed to not lose money.) The two pieces of good news are: (i) Proposition 4.1 shows that the loss is bounded by an absolute constant (depending on $|X|$, which is often 2), independent of the number of players; (ii) if the information gathered is valuable, it's reasonable to expect to pay something for it.[7] So in many cases this loss is palatable and it makes sense to run a prediction market.

4.4 Strategic Issues

The next proposition is a weak form of truthfulness.

Proposition 4.2 *Suppose that either:*

- *players are myopic, meaning they do not account for the future ramifications of their actions; or*
- *each player trades exactly once, in a fixed order.*

Then, the unique best response of every player is to report her true beliefs.

Proposition 4.2 follows immediately from the payoff structure in (1): a player cannot affect the second term of her payoff, and she maximizes the first term by reporting her true beliefs (since S is a strictly proper scoring rule).

Strategic issues can arise in market scoring rules with non-myopic players, when players can report multiple times, if one player's report can cause other players to update their beliefs. Intuitively, an early misreport by one player can cause other players to misprice the security, to the later benefit of the misreporting player. For example, suppose two fair coins are flipped, with Alice seeing the outcome of the first coin and Bob the second coin. Consider a security that pays \$1 if both coins come up heads. Suppose the first coin is "tails" and Alice goes first. If she reports truthfully (i.e., 0% chance of the relevant event), then Bob will do nothing. But suppose Alice misleads Bob by reporting the opposite (i.e., 50/50). If Bob takes this report at face value (i.e., assumes that Alice saw "heads"), and his own coin was "heads," then he will respond by reporting 100%. But then Alice can report the correct distribution (0%), procuring a big payout from the market scoring rule (at Bob's expense).

Recent research has identified some conditions on players' information such that it is an equilibrium for all players to report immediately and truthfully [2, 4].

4.5 The Logarithmic Market Scoring Rule (LMSR)

The special case of a market scoring rule where S is taken as the logarithmic scoring rule—that is, $S(\mathbf{p}, i) = \ln p_i$, possibly shifted and/or scaled—is called (wait for it) the *logarithmic market scoring rule (LMSR)* [5]. Internal corporate prediction markets often use the LMSR, and much of the research literature on prediction markets concerns the LMSR.

[7] Also, thinking back to the case of a single player, it is intuitively clear that we'll need to pay something for her information.

5 Automated Market Makers

5.1 Preliminaries

For someone already familiar with scoring rules, market scoring rules are natural and their financial loss and incentive properties are transparent (Propositions 4.1 and 4.2). But what if we want to replicate the "look and feel" of continuous double auctions, with players buying and trading shares, rather than reporting probability distributions?

It turns out that market scoring rules can be implemented as automated market makers, with players implicitly changing the market's prediction by buying and selling shares. We illustrate this reimplementation for the special case of the LMSR.

Since an automated market-maker accepts any buy or sell offer at the current market price, defining the market boils down to defining what the current market prices are (as a function of what's happened in the past). We consider a set X of k possible outcomes, with one security for each outcome.

We use $\pi_i(\mathbf{q})$ to denote the current market price of the ith security, given that a net q_j shares of the jth security have been sold (here $\mathbf{q}$ denotes the quantity vector $(q_1, \ldots, q_k)$). Our definition of π_i will ensure that the following two constraints are met:

$$\pi_i(\mathbf{q}) \geq 0 \tag{2}$$

for all $i \in \{1, 2, \ldots, k\}$ and $\mathbf{q} \geq 0$, and

$$\sum_{i=1}^{k} \pi_i(\mathbf{q}) = 1 \tag{3}$$

for all $\mathbf{q} \geq 0$. These two constraints ensure that, at any time, the market has a well-defined "current belief" about which outcome will occur, with the price of a security indicating the probability of the corresponding outcome.

Suppose a player executes the trade $\mathbf{z} = (z_1, \ldots, z_k)$, with positive z_i's denoting buy orders and negative z_i's denoting sell orders. (A player is not allowed to sell more shares of a security than what she already owns.) The total price of this trade is, by definition,

$$\int_0^{\mathbf{z}} \sum_{i=1}^{k} \pi_i(\mathbf{q} + \mathbf{x}) d\mathbf{x}. \tag{4}$$

Some comments. First, note that the current prices π are not "locked in"—as soon as a player buys or sells some shares, the current price changes. So the total price is computed by integrating over the trajectory taken by the market prices as the trade is executed. Second, the integral in (4) is a path integral in $\mathbb{R}^k$, not a univariate integral. And path integrals generally depend on the choice of the path (not just on its endpoints). Path-dependence would be a non-starter for us, since then one could make a trade (broken into a sequence of mini-trades in a particular way) and then the reverse trade (broken up into mini-trades in a different way) and make money—a "money pump," in effect. To ensure path independence, we will choose the pricing function π so that it is the gradient ∇C of some "cost function" $C : \mathbb{R}^k \to \mathbb{R}$. Then, the integral in (4) is simply

$$C(\mathbf{q}+\mathbf{z}) - C(\mathbf{q}). \tag{5}$$

5.2 Details

The story so far: defining a trading-based prediction market boils down to defining the current market prices π, which in turn boils down to defining the cost function C (and then setting $\pi = \nabla C$). So how do we choose C (and hence π) so that the incentives faced by participants are the same as with the LMSR? There is a systematic way of answering this question (for any market scoring rule, not just the LMSR) [3], but for brevity we'll use the "guess and check" method, pulling the appropriate cost function out of a hat.

We define

$$C(\mathbf{q}) = \ln\left(\sum_{i=1}^{k} e^{q_i}\right) \tag{6}$$

and hence (using the chain rule)

$$\pi_i(\mathbf{q}) = \frac{\partial}{\partial q_i} C(\mathbf{q}) = \frac{e^{q_i}}{\sum_{j=1}^{k} e^{q_j}}. \tag{7}$$

The final description of the market is very simple, and doesn't require remembering any multivariate calculus: just use the formula in (7) to keep track of the market prices, and accept any trades (buying or selling) at the current prices, with the total price of a trade given by (5) and (6).

As promised, the π_i's always satisfy the constraints (2) and (3) and hence can be interpreted as a probability distribution over the outcomes X. Note that, up to a discretization error, a player can move the "market beliefs" $\pi_1(\mathbf{q}), \ldots, \pi_k(\mathbf{q})$ to any distribution that she wants by buying and selling shares appropriately.[8] That is, a player can simulate a belief report to a market scoring rule by executing a suitable trade.

The following is the formal statement of equivalence between the LMSR and the market with prices defined as in (7).

Theorem 5.1 *For every quantity vector* $\mathbf{q}$*, trade* $\mathbf{z}$ *with* $\mathbf{q}+\mathbf{z} \geq 0$*, and outcome* $i \in X$*,*

$$\text{profit from trade } \mathbf{q} \mapsto \mathbf{q}+\mathbf{z} = \text{profit from update } \pi(\mathbf{q}) \mapsto \pi(\mathbf{q}+\mathbf{z}) \text{ under LMSR.} \tag{8}$$

That is, when a player makes a trade in this market, the consequence for the player is exactly as if she reported the corresponding probability distribution $\pi(\mathbf{q}+\mathbf{z})$ to the LMSR. This means that the loss and incentive properties for the LMSR (Proposition 4.1 and 4.2)

[8] The prices never reach 0, but they can get arbitrarily close.

hold unchanged for a market with prices defined as in (7) (modulo the discretization error mentioned earlier).[9]

Proof of Theorem 5.1: Suppose the realized outcome is $i \in X$. Recalling (5), the left-hand side of (8) is

$$\underbrace{z_i}_{\text{payoff of new shares}} - \underbrace{(C(\mathbf{q}+\mathbf{z}) - C(\mathbf{q}))}_{\text{cost of trade}},$$

which by (6) is

$$z_i - \left[\ln\left(\sum_{j \in X} e^{q_j + z_j}\right) - \ln\left(\sum_{j \in X} e^{q_j}\right)\right].$$

Meanwhile, the right-hand side of (8) is

$$\begin{aligned}
S(\pi_i(\mathbf{q}+\mathbf{z}), i) - S(\pi_i(\mathbf{q}), i) &= \ln\left(\frac{e^{q_i+z_i}}{\sum_{j \in X} e^{q_j+z_j}}\right) - \ln\left(\frac{e^{q_i}}{\sum_{j \in X} e^{q_j}}\right) \\
&= (q_i + z_i) - \ln\left(\sum_{j \in X} e^{q_j+z_j}\right) - q_i + \ln\left(\sum_{j \in X} e^{q_j}\right) \\
&= z_i - \left[\ln\left(\sum_{j \in X} e^{q_j+z_j}\right) - \ln\left(\sum_{j \in X} e^{q_j}\right)\right],
\end{aligned}$$

the same number as the left-hand side. ■

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[9] It can also be useful to work with a scaled version of the logarithmic scoring rule, with $S(\mathbf{p}, i) = b \ln p_i$ for a parameter $b > 0$. In the corresponding prices (7), all of the exponents change from q_i to q_i/b. (The cost function (6) becomes $C(\mathbf{q}) = b \ln(\sum_{i=1}^{k} e^{q_i/b})$.) Thus, as b grows larger, the market becomes more liquid, in the sense that the trade of a single share has a smaller effect on the current market price. In practice one wants a b that is neither too big nor too small, so that prices move with reasonable-sized trades, but not too much. Choosing a good value of b generally requires trial and error. One drawback of larger b's is that the bound on the market's loss (Proposition 4.1) degrades as $b \ln |X|$.

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