

CS269I: Exercise Set #9

Due by 11:59 PM on Wednesday, December 5, 2018

Instructions:

- (1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.
- (2) Submission instructions: We are using Gradescope for the homework submissions. Go to www.gradescope.com to either login or create a new account. Use the course code MZZ2BV to register for CS269I. Only one person needs to submit the assignment. When submitting, please remember to add your partner's name (if any) in Gradescope.
- (3) Please type your solutions if possible. We encourage you to use the LaTeX template provided on the course home page.
- (4) Write convincingly but not excessively. You should be able to fit all of your solutions into two pages, if not less.
- (5) Except where otherwise noted, you may refer to the course lecture notes and the specific supplementary readings listed on the course Web page *only*.
- (6) You can discuss the exercises verbally at a high level with other groups. And of course, you are encouraged to contact the course staff (via Piazza or office hours) for additional help.
- (7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.
- (8) No late assignments will be accepted, but we will drop your lowest exercise set score.

Lecture 17 Exercises

Exercise 30

For exercises 30–33, assume that there is an odd number of voters. For a set of votes (i.e., ranked lists) over a set A of alternatives, we say that alternative a *beats* b if more than half of the voters rank a somewhere above b in their lists. A *Condorcet winner* is an alternative that beats every other alternative.

Show by example that there is not always a Condorcet winner.

Exercise 31

A voting rule satisfies the *Condorcet condition* if it elects a Condorcet winner whenever one exists.

Does the plurality rule satisfy the Condorcet condition?¹ Provide either a proof that it does or a counterexample (i.e., a set of votes where there is a Condorcet winner a and the rule chooses an alternative different from a).

Exercise 32

Does ranked-choice voting satisfy the Condorcet condition? Provide either a proof that it does or a counterexample.

¹Throughout this exercise set, assume that ties are broken in some consistent way, such as lexicographically.

Exercise 33

Does the Borda count satisfy the Condorcet condition? Provide either a proof that it does or a counterexample.

Lecture 18 Exercises

Exercise 34

Recall the model of fair division from lecture. Prove that, for two players, every proportional allocation is also envy-free.

Exercise 35

Show that, with three players, a proportional allocation need not be envy-free.