Abstract

The incentive-compatibility (IC) properties of blockchain transaction fee mechanisms (TFMs) have been investigated with passive block producers that are motivated purely by the net rewards earned at the consensus layer. This paper introduces a model of active block producers that have their own private valuations for blocks (representing, for example, value derived from the application layer). The block producer surplus in our model can be interpreted as a common interpretation of the phrase “maximal extractable value (MEV).”

We prove that TFM design is fundamentally more difficult with active block producers than with passive ones: With active block producers, no non-trivial or approximately welfare-maximizing TFM can be IC for both users and block producers. These impossibility results can be interpreted as a mathematical justification for augmenting TFMs with additional components.

We proceed to a more fine-grained model inspired by current practice, in which we distinguish the roles of “searchers” (who identify opportunities for value extraction) and “proposers” (who participate directly in consensus and choose the final published block). Here, we first consider a TFM that resembles the block production process in practice where each transaction is effectively sold to a searcher in a first-price auction. We then explore the design space more generally and design a mechanism that circumvents our previous impossibility results. Our mechanism (“SAKA”) is deterministic, IC (for users, searchers, and the block producer), sybil-proof, and guarantees roughly 50% of the maximum-possible welfare when transaction sizes are small relative to block sizes. We conclude with a matching negative result: even when transactions are small relative to blocks, no IC, sybil-proof, and deterministic TFM can guarantee more than 50% of the maximum-possible welfare.

1 Introduction

1.1 Transaction Fee Mechanisms for Allocating Blockspace

Blockchain protocols such as Bitcoin and Ethereum process transactions submitted by users, with each transaction advancing the “state” of the protocol (e.g., the set of Bitcoin UTXOs, or the state...
of the Ethereum Virtual Machine). Such protocols have finite processing power, so when demand for transaction processing exceeds the available supply, a strict subset of the submitted transactions must be chosen for processing. To encourage the selection of the “most valuable” transactions, the transactions chosen for processing are typically charged a transaction fee. The component of a blockchain protocol responsible for choosing the transactions to process and what to charge for them is called its transaction fee mechanism (TFM).

Previous academic work on TFM design (surveyed in Section 1.5) has focused on the game-theoretic properties of different designs, such as incentive-compatibility from the perspective of users (ideally, with a user motivated to bid its true value for the execution of its transaction), of block producers (ideally, with a block producer motivated to select transactions to process as suggested by the TFM), and of cartels of users and/or block producers. Discussing incentive-compatibility requires defining utility functions for the relevant participants. In most previous works on TFM design (and in this paper), users are modeled as having a private value for transaction inclusion and a quasi-linear utility function (i.e., value enjoyed minus price paid). In previous work—and, crucially, unlike in this work—a block producer was modeled as passive, meaning its utility function was the net reward earned (canonically, the unburned portion of the transaction fees paid by users, possibly plus a block reward).

While this model is a natural one for the initial investigation of the basic properties of TFMs, it effectively assumes that block producers are unaware of or unconcerned with the semantics of the transactions that they process—that there is a clean separation between users (who have value only for activity at the application layer) and block producers (who, if passive, care only about payments received at the consensus layer).

1.2 MEV and Active Block Producers

It is now commonly accepted that, at least for blockchain protocols that support a decentralized finance (“DeFi”) ecosystem, there are unavoidable interactions between the consensus layer (block producers) and the application layer (users), and specifically with block producers deriving value from the application layer that depends on which transactions they choose to process (and in which order). For a canonical example, consider a transaction that executes a trade on an automated market maker (AMM), exchanging one type of token for another (e.g., USDC for ETH). The spot price of a typical AMM moves with every trade, so by executing such a transaction, a block producer may move the AMM’s spot price out of line with the external market (e.g., on centralized exchanges (CEXs) like Coinbase), thereby opening up an arbitrage opportunity (e.g., buying ETH on a CEX at the going market price and then selling it on an AMM with a larger spot price). The block producer is uniquely positioned to capture this arbitrage opportunity, by executing its own “backrunning” transaction (i.e., a trade in the opposite direction) immediately after the submitted trade transaction.

The first goal of this paper is to generalize the existing models of TFM design in the minimal way that accommodates active block producers, meaning block producers with a utility function that depends on both the transactions in a block and the net fees earned. Specifically, in addition to the standard private valuations for transaction inclusion possessed by users, the block producer will have its own private valuation, which is an abstract function of the block that it publishes. We then assume that a block producer acts to maximize its block producer surplus (BPS), meaning its private value for the published block plus any additional profits (or losses) from fees (or burns). In the interests of a simple but general model, we deliberately avoid microfounding the private
valuation function of a block producer or committing to any specifics of the application layer. Our model captures, in particular, canonical on-chain DeFi opportunities such as arbitrage and liquidation opportunities, but a block producer’s valuation can reflect arbitrary preferences, perhaps derived also from off-chain activities (e.g., a bet with a friend that settles on-chain) or subjective considerations.

The extraction of application-layer value by block producers, in DeFi and more generally, was first studied by Daian et al. [18] under the name “MEV” (for “maximal extractable value”). At this point, the term has transcended any specific definition—in both the literature and popular discourse, it is used, often informally, to refer to a number of related but different concepts. We argue that our definition of BPS captures, in a precise way and in a concrete economic model, one of the more common colloquial meanings of the term “MEV.”

1.3 The Block Production Supply-Chain

In the first part of this paper, we treat a block producer as a single entity that publishes a block based on the transactions that it is aware of. This would be an accurate model of block production, as carried out by miners in proof-of-work protocols and validators in proof-of-stake protocols, up until a few years ago. More recently, especially in the Ethereum ecosystem, block production has evolved into a more complex process, typically involving “searchers” (who identify opportunities for extraction from the application layer), “builders” (who assemble such opportunities into a valid block), “relays” (who gather blocks from builders and select the most profitable one for the proposer), and “proposers” (who participate directly in the blockchain protocol and make the final choice of the published block), among others. One interpretation of a block producer in our basic model is as a vertically integrated party that performs the jobs of all of these entities.

In the second part of the paper, we consider a more fine-grained model of the block production process, in which the role of finding MEV extraction opportunities is decoupled from the proposer’s role of participating in consensus and is instead performed by specialized searchers. An interpretation of this model is that the proposer runs an open-source consensus client to collect block rewards, while outsourcing the complicated task of finding MEV opportunities to searchers. This is in the same spirit as mev-geth, which was a widely-used Ethereum client written by Flashbots that proposers could run to allow for the submission of both regular transactions by users and wrapped bundles of transactions by searchers.\footnote{See \url{https://github.com/flashbots/mev-geth/blob/master/README.md}} Prior to mev-geth, searchers and users were treated equally by proposers and competed with each other for inclusion; among other issues, multiple searchers pursuing the same MEV extraction opportunity would often have their extraction transactions included in a block, with the first such transaction capturing the opportunity and the rest failing (but still paying transaction fees for inclusion and wasting valuable blockspace). Mev-geth introduced an explicit auction, upstream from the blockchain’s fee mechanism, in which searchers could compete directly with each other to capture MEV extraction opportunities. Our model can be viewed as formalizing this idea by allowing a TFM to treat searchers and users differently, subject to different rules for inclusion and payment.

1.4 Overview of Results

Our starting point is the model for transaction fee mechanism design defined in [47]. In this model, each user has a private valuation for the inclusion of a transaction in a block, and submits a bid
along with its transaction. As in [47], we consider TFMs that choose the included transactions and payments based solely on the bids of the pending transactions (as opposed to, say, based also on something derived from the semantics of those transactions). A block producer publishes any block that it wants, subject to feasibility (e.g., with the total size of the included transactions respecting some maximum block size). A TFM is said to be dominant-strategy incentive-compatible (DSIC) if every user has a dominant (i.e., always-optimal) bidding strategy. The DSIC property is often associated with a good “user experience (UX),” in the sense that each user has an obvious optimal bid. In [47], a TFM was said to be incentive-compatible for myopic miners (MMIC) if it expects a block producer to publish a block that maximizes the net fees earned (at the consensus layer). Here, we introduce an analogous definition that accommodates active block producers: We call a TFM incentive-compatible for block producers (BPIC) if it expects a block producer to publish a block that maximizes its private valuation plus the net fees earned. An ideal TFM would satisfy, among other properties, both DSIC and BPIC.

1.4.1 Vertically Integrated Active Block Producers

We begin with a model in which there are only users and a single (vertically integrated) active block producer, and show that there are fundamental barriers to designing ideal transaction fee mechanisms in this case.

Our first result (Theorem 3.1) is a proof that with active block producers no non-trivial TFM satisfies both DSIC and BPIC, where “non-trivial” means that users must at least in some cases pay a nonzero amount for transaction inclusion. (In contrast, with passive block producers and no MEV, the “tipless mechanism” suggested in [47] is non-trivial and satisfies both DSIC and BPIC (see also Example 2.12); the same is true of the EIP-1559 mechanism of Buterin et al. [12] (see Example 2.11), provided the mechanism’s base fee is not excessively low [47].) In particular, the EIP-1559 and tipless mechanisms fail to satisfy DSIC and BPIC when block producers can be active. Intuitively, for these mechanisms, a user might be motivated to underbid in the hopes of receiving an effective subsidy by the block producer (who may include the transaction anyways, if it derives outside value from it).

Our second result (Theorem 3.3) formalizes the intuition that TFMs that do not charge non-zero transaction fees—and in particular (by Theorem 3.1), TFMs that are both DSIC and BPIC—cannot guarantee any approximation of the maximum-possible social welfare. Intuitively, the issue is the lack of alignment between the preferences of users and of the block producer: If a block producer earns no transaction fees from any block, it might choose a block with non-zero private value but only very low-value transactions over one with no private value but very high-value transactions.

1.4.2 TFMs with Competing Searchers

We then consider a more fine-grained model of block production that is inspired by current practice, in which we distinguish the roles of “searchers” (who actively identify opportunities for value extraction from the application layer and compete for the right to take advantage of them) and “proposers” (who participate directly in the blockchain protocol and make the final choice of the published block). Searchers can effectively act as an “MEV oracle” for a transaction fee mechanism, thereby enlarging the mechanism design space.

In this model, we first consider a TFM that resembles how searchers have traditionally been incorporated into the block production process, and specifically mev-geth (see Section 2.5). Intu-
itive, this mechanism runs a first-price auction for each transaction among the interested searchers; the winning bid then acts as an estimate of the transaction’s MEV, which the TFM can then use to charge prices in a way that recovers the DSIC property for users (Theorem 4.2).

We then explore the TFM design space with searchers more generally, with a focus on good approximate welfare guarantees. Our main contribution here is what we call the SAKA mechanism, which is deterministic, DSIC for users, DSIC for searchers, BPIC, and sybil-proof, and which guarantees roughly 50% of the maximum-possible welfare when transaction sizes are small relative to block sizes (as they are in practice); see Theorems 5.3 and 5.5. In particular, this combination of guarantees shows that TFMs with searchers can evade impossibility results that apply to TFMs without searchers (such as Theorem 3.3). We further show in Theorem 5.7 that, even when transactions are small relative to blocks, no deterministic, DSIC, and sybil-proof TFM can guarantee more than 50% of the maximum-possible welfare. (By “sybil-proof,” we mean that no user or searcher can ever profit from creating additional user or searcher identities and submitting fake transactions or bundles under those identities.)

1.5 Related Work

Defining MEV. Daian et al. [18] introduced the notion of miner/maximal extractable value. They defined MEV as the value that miners or validators could obtain by manipulating the transactions in a block. Since this work there have been many follow-up works attempting to formalize MEV and analyze its effects in both theory and practice. Attempts to give exact theoretical characterizations of MEV appear in [48, 41, 9, 5]. Broadly, these works define MEV by defining sets of valid transaction sequences and allowing the block producer to maximize their value over these sequences. These definitions are very general, but in exchange have to this point proved analytically intractable. Several empirical papers study the impact and magnitude of MEV using heuristics applied to on-chain data [43, 44, 50]. Another line of work [32, 28, 8] studies MEV in specific contexts, such as for arbitrage in AMMs, in which it is possible to characterize how much MEV can be realized from certain transactions. In particular, Kulkarni et al. [32] give formal statements on how, under different AMM designs, MEV affects the social welfare of the overall system.

General TFM literature. The model in this paper is closest to the one used by Roughgarden [47] to analyze (with passive block producers) the economic properties of the EIP-1559 mechanism [12], the TFM used currently in the Ethereum blockchain.

Precursors to that work (also with passive block producers) include studies of a “monopolistic price” transaction fee mechanism [31, 53] (also considered recently by Nisan [40]), and work of Basu et al. [10] that proposed a more sophisticated variant of that mechanism. There have also been several follow-up works to [47] that use similar models (again, with passive block producers). Chung and Shi [17], Chung et al. [16], and Gafni and Yaish [26] proved impossibility results showing that the incentive-compatible guarantees of the EIP-1559 mechanism are in some respects the best possible. There have also been attempts to circumvent such impossibility results by relaxing the notion of incentive compatibility [17, 25], using cryptography [49], considering a Bayesian setting [55], or mixtures of these ideas [51]. Other recent works [21, 95] study the dynamics of the base fee in the EIP-1559 mechanism.

MEV-aware mechanism design. There has been much interest among both researchers and practitioners in restructuring the block production supply chain to address MEV [32, 28]. On the
academic side, the bulk of these approaches involve cryptographic techniques \cite{31, 36, 54, 11} or changes at the consensus layer \cite{30, 29, 13, 33}. Relatively recently, there have been some initial studies on the impact of economic mechanisms for mitigating MEV such as order-flow auctions \cite{27} and mev-boost \cite{2}; see \cite{42, 45, 7}. In practice, to this point, economic approaches to addressing MEV have been more popular than cryptographic ones; examples include, among others, mev-share \cite{38}, UniswapX \cite{3}, and MEV Blocker \cite{1}. The model in this paper integrates some of the ideas behind these deployed applications into the existing mathematical frameworks for the design and analysis of transaction fee mechanisms.

**Impossibility results in mechanism design.** The impossibility results in Section 3 may appear superficially related to other such results in mechanism design. For example, the classic Myerson-Satterwhaite Theorem \cite{39} states that there is no efficient, individually rational, Bayesian incentive compatible, and budget-balanced mechanism for bilateral trade. Fundamentally, the Myerson-Satterwhaite Theorem is driven by the tension between welfare and budget-balance in the presence of incentive-compatibility constraints on the participants. Our main impossibility result (Theorem 3.1), meanwhile, is driven by the combination of incentive-compatibility constraints for users (analogous to the usual participants of a mechanism design problem) and also such a constraint for a self-interested party that is tasked with carrying out the allocation rule of the mechanism (the block producer). As such, our setup more closely resembles that of credible mechanisms (discussed below) than more traditional mechanism design settings. In particular, Theorem 3.1 holds even in the absence of any welfare-maximization or exact budget-balance requirements (a non-zero burning rule in the sense of Section 2.3 is tantamount to relaxing budget-balance).

**Credible mechanisms.** Akbarpour and Li \cite{4} introduce the notion of credible mechanisms, where any profitable deviations by the auctioneer can be detected by at least one user. While similar in spirit to the concept of BPIC introduced here (and the special case of MMIC introduced in \cite{47}), there are several important differences. For example, the theory of credible mechanisms assumes fully private communication between bidders and the auctioneer and no communication among bidders, whereas TFM bids are commonly collected from a public mempool. Another difference is that a block producer in our model can manipulate only the allocation rule of a mechanism (as the payment and burning rules are enforced by the code of the blockchain protocol), while in the credible mechanisms framework the auctioneer can also manipulate the payment rule. In a different direction, there is also a line of follow-up work that takes advantage of cryptographic primitives to build credible auctions on the blockchain \cite{23, 20, 15, 22}.

### 2 Model

This section defines transaction fee mechanisms, the players and their objectives, and the relevant incentive-compatibility notions. Sections 2.1-2.4 describe the basic model (with vertically integrated, active block producers) that is considered in Section 3. Section 2.5 augments this model with searchers, which play a central role in Sections 4 and 5.
2.1 The Players and Their Objectives

Users. Users submit transactions to the blockchain protocol. The execution of a transaction updates the state of the protocol (e.g., users’ account balances). The rules of the protocol specify whether a given transaction is valid (e.g., whether it is accompanied by the required cryptographic signatures). From now on, we assume that all transactions under consideration are valid. Every transaction $t$ has a publicly known size $s_t$ (e.g., the gas limit of an Ethereum transaction).

We assume that each user submits a single transaction $t$ and has a nonnegative valuation $v_t$, denominated in a base currency like USD or ETH, for its execution in the next block. This valuation is private, in the sense that it is initially unknown to all other parties. We assume that the utility function of each user—the function that the user acts to maximize—is quasi-linear, meaning that its utility is either 0 (if its transaction is not included in the next block) or $v_t - p$ (if its transaction is included and it must pay a fee of $p$).

Blocks. A block is a finite set of transactions. A feasible block is a block that respects any additional constraints imposed by the protocol. For example, if the protocol specifies a maximum block size, then feasible blocks might be defined as those that comprise only valid transactions and also respect the block size limit.

Block producers (BPs). We consider blockchain protocols for which the contents of each block are selected by a single entity, which we call the block producer (BP). We focus on the decision-making of the BP that has been chosen at a particular moment in time (perhaps using a proof-of-work or proof-of-stake-based lottery) to produce the next block. We assume that whatever block the BP chooses is in fact published, with all the included transactions finalized and executed.

A BP chooses a block $B$ from some abstract non-empty set $\mathcal{B}$ of feasible blocks, called its blockset. For example, the set $\mathcal{B}$ might consist of all the feasible blocks that comprise only transactions that the BP knows about (perhaps from a public mempool, or perhaps from private communications) along with transactions that the BP is in a position to create itself (e.g., a backrunning transaction). As with users, we model the preferences of a BP with a quasi-linear utility function, meaning the difference between its private value for a block (again, denominated in a base currency like USD or ETH) minus the (possibly negative) payment that it must make. Unlike with users, to avoid modeling any details of why a BP might value a block (e.g., due to the extraction of value from the application layer), we allow a BP to have essentially arbitrary preferences over blocks. More formally, we assume that a BP has a private valuation that is an arbitrary (real-valued) function $v_{BP}$ over blocks, and the BP acts to maximize its block producer surplus (BPS):

\[
\underbrace{v_{BP}(B) + \text{net fees earned}}_{\text{block producer surplus (BPS)}}.
\]

Holders. The final category of participants, which are non-strategic in our model but relevant for our definition of welfare in Section 2.2, is the holders of the blockchain protocol’s native currency. As we’ll see in Section 2.3, TFMs are in a position to mint or burn this currency, which corresponds to inflation or deflation, respectively. We treat TFM mints and burns as transfers from and to, respectively, the existing holders of this currency. Formally, we define the collective utility function of currency holders to be the net amount of currency burned by a TFM.
2.2 Welfare

According to the principle of welfare-maximization, a scarce resource like blockspace should be allocated to maximize the total utility of all the “relevant participants,” which in our case includes the users, the BP, and the currency holders. Because all parties have quasi-linear utility functions and all TFM transfers will be between members of this group (from users to the BP, from the BP to holders, etc.), the welfare of a block is simply the sum of the user and BP valuations for it:

\[ W(B) := v_{BP}(B) + \sum_{t \in B} v_{t} \]  

(1)

Holders are assumed to be passive and thus have no valuations to contribute to the sum.

2.3 Transaction Fee Mechanisms

The outcome of a transaction fee mechanism is a block to publish and a set of transfers (user payments, burns, etc.) that will be made upon the block’s publication. In line with the preceding literature on TFMs and the currently deployed TFM designs, we assume that each user that creates a transaction \( t \) submits along with it a nonnegative bid \( b_{t} \) (i.e., willingness to pay), and that a TFM bases its transfers on the set of available transactions and the corresponding bids. (The BP submits nothing to the TFM.) A TFM is defined primarily by its payment and burning rules, which specify the fees paid by users and the burned funds implicitly received by holders (with the BP pocketing the difference).

**Payment and burning rules.** The payment rule specifies the payments made by users in exchange for transaction inclusion.

**Definition 2.1 (Payment Rule)** A payment rule is a function \( p \) that specifies a nonnegative payment \( p_{t}(B, b) \) for each transaction \( t \in B \) in a block \( B \), given the bids \( b \) of all known transactions.

The value of \( p_{t}(B, b) \) indicates the payment from the creator of an included transaction \( t \in B \) to the BP that published that block. (Or, if the rule is randomized, the expected payment.) We consider only individually rational payment rules, meaning that \( p_{t}(B, b) \leq b_{t} \) for every included transaction \( t \in B \). We can interpret \( p_{t}(B, b) \) as 0 whenever \( t / \in B \). Finally, we assume that every creator of an included transaction has the funds available to pay its full bid, if necessary (otherwise, the block \( B \) should be considered infeasible).

The burning rule specifies how much money must be burned by a BP along with the publication of a given block.

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2 We stress that the welfare of a block measures the “size of the pie” and says nothing about how this welfare might be split between users, the BP, and holders (i.e., about the size of each slice). Distributional considerations are important, of course, but they are outside the scope of this paper.

3 We assume that users and BPs are risk-neutral when interacting with a randomized TFM.

4 This differs superficially from the formalism in [47], in which a burning rule specifies per-transaction (rather than per-block) transfers from users (rather than the BP) to currency holders. The payment rule here can be interpreted as the sum of the payment and burning rules in [47], and the per-block burning rule here can be interpreted as the sum of the burns of a block’s transactions in [47].
**Definition 2.2 (Burning Rule)** A *burning rule* is a function $q$ that specifies a nonnegative burn $q(B, b)$ for a block $B$, given the bids $b$ of all known transactions.

The value of $q(B, b)$ indicates the amount of money burned (i.e., paid to currency holders) by the BP upon publication of the block $B$. (Or, if the rule is randomized, the expected amount.) We assume that, after receiving users’ payments for the block, the BP has sufficient funds to pay the burn required of the block that it publishes (otherwise, the block $B$ should be considered infeasible). We stress that the payment and burning rules of a TFM are hard-wired into a blockchain protocol as part of its code. This is why their arguments—the transactions chosen for execution and their bids, and perhaps (as in [17]) the bids of some additional, not-to-be-executed transactions—must be publicly recorded as part of the blockchain’s history. (E.g., late arrivals should be able to reconstruct users’ balances, including any payments dictated by a TFM, from this history.) A BP cannot manipulate the payment and burning rules of a TFM, except inasmuch as it can choose which block $B \in \mathcal{B}$ to publish.

**Allocation rules.** In our model, a BP has unilateral control over the block that it chooses to publish. Thus, a TFM’s allocation rule—which specifies the block that should be published, given all of the relevant information—can only be viewed as a recommendation to a BP. Because the (suggested) allocation rule would be carried out by the BP and not by the TFM directly, it can sensibly depend on arguments not known to the TFM (but known to the BP), specifically the BP’s valuation $v_{BP}$ and blockset $\mathcal{B}$.

**Definition 2.3 (Allocation Rule)** An *allocation rule* is a function $x$ that specifies a block $x(b, v_{BP}, \mathcal{B}) \in \mathcal{B}$, given the bids $b$ of all known transactions, the BP valuation $v_{BP}$, and the BP blockset $\mathcal{B}$.

An allocation rule $x$ induces per-transaction allocation rules with, for a transaction $t$, $x_t(b, v_{BP}, \mathcal{B}) = 1$ if $t \in x(b, v_{BP}, \mathcal{B})$ and 0 otherwise.

**Definition 2.4 (Transaction Fee Mechanism (TFM))** A *transaction fee mechanism (TFM)* is a triple $(x, p, q)$ in which $x$ is a (suggested) allocation rule, $p$ is a payment rule, and $q$ is a burning rule.

A TFM is defined relative to a specific block publishing opportunity. A blockchain protocol is free to use different TFMs for different blocks (e.g., with different base fees), perhaps informed by the blockchain’s past history.

**Utility functions and BPS revisited.** With Definitions 2.1–2.4 in place, we can express more precisely the strategy spaces and utility functions introduced in Section 2.1. We begin with an expression for the utility of a user (as a function of its bid) for a TFM’s outcome, under the assumption that the BP always chooses the block suggested by the TFM’s allocation rule.

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5 An alternative to money-burning that has similar game-theoretic and welfare properties is to transfer $q(B, b)$ to stakeholders other than the BP, such as a foundation or the producers of future blocks.
Definition 2.5 (User Utility Function) For a TFM \((x, p, q)\), BP valuation \(v_{BP}\), BP blockset \(B\), and bids \(b_{-t}\) of other transactions, the utility of the originator of a transaction \(t\) with valuation \(v_t\) and bid \(b_t\) is

\[
u_t(b_t) := v_t(x_t((b_t, b_{-t}), v_{BP}, B) - p_t(B, (b_t, b_{-t}))),
\]

where \(B := x_t((b_t, b_{-t}), v_{BP}, B)\).

In (2), we highlight the dependence of the utility function on the argument that is directly under a user’s control, the bid \(b_t\) submitted with its transaction.

The BP’s utility function, the block producer surplus, is then:

Definition 2.6 (Block Producer Surplus (BPS)) For a TFM \((x, p, q)\), BP valuation \(v_{BP}\), BP blockset \(B\), and transaction bids \(b\), the block producer surplus of a BP that chooses the block \(B \in B\) is

\[
u_{BP}(B) := v_{BP}(B) + \sum_{t \in B} p_t(B, b) - q(B, b).
\]

In (3), we highlight the dependence of the BP’s utility function on the argument that is under its direct control, its choice of a block. The BP’s utility depends on the payment and burning rules of the TFM, but not on its allocation rule (which the BP is free to ignore, if desired).

Finally, the collective utility function of (passive) currency holders for a block \(B\) with transaction bids \(b\) is \(q(B, b)\), the amount of currency burned by the BP. (As promised, for a block \(B\), no matter what the bids and the TFM, the sum of the utilities of users, the BP, and holders is exactly the welfare defined in (1).)

2.4 Incentive-Compatible TFMs

In this paper, we focus on two incentive-compatibility notions for TFMs—which, as we’ll see, are already largely incompatible—one for users and one for block producers. We begin with the latter.

BPIC TFMs. We assume that a BP will choose a block to maximize its utility function, the BPS (Definition 2.6). The defining equation (3) shows that, once the payment and burning rules of a TFM are fixed, a BP’s valuation and blockset dictate the unique (up to tie-breaking) BPS-maximizing block for each bid vector. We call an allocation rule consonant if, given the payment and burning rules, it instructs a BP to always choose such a block (breaking ties in an arbitrary but consistent fashion).

Definition 2.7 (Consonant Allocation Rule) An allocation rule \(x\) is consonant with the payment and burning rules \(p\) and \(q\) if:

(a) for every BP valuation \(v_{BP}\) and blockset \(B\), and for every choice of transaction bids \(b\),

\[
x(b, v_{BP}, B) \in \arg \max_{B \in B} \left\{ v_{BP}(B) + \sum_{t \in B} p_t(B, b) - q(B, b) \right\};
\]

(b) for some fixed total ordering on the blocks of \(B\), the rule breaks ties between BPS-maximizing blocks according to this ordering.
Because a BP can see all bids after they are submitted, they can also insert their own “fake” transactions along with “shill” bids for them (e.g., to manipulate the payment or burning rules of the TFM); we require that a BP is never incentivized to include such shill bids.

**Definition 2.8 (Shill-Proof)** Payment and burning rules \( p \) and \( q \) are shill-proof if for every set \( T \) of user-submitted transactions with bids \( b \), every feasible block \( B \subseteq T \), every set \( F \) of BP-submitted fake transactions with bids \( b_F \), and every feasible block \( B \cup S \) with \( S \subseteq F \),

\[
\left( \sum_{t \in B} p_t(B, b) \right) - q(B, b) \geq \left( \sum_{t \in B} p_t(B \cup S, (b, b_F)) \right) - q(B \cup S, (b, b_F)) .
\]

**BPIC** TFMs are then precisely those that are shill-proof and always instruct a BP to choose a BPS-maximizing block (breaking ties consistently).

**Definition 2.9 (Incentive-Compatibility for Block Producers (BPIC))** A TFM \((x, p, q)\) is incentive-compatible for block producers (BPIC) if:

(a) \( x \) is consonant with \( p \) and \( q \);

(b) \( p \) and \( q \) are shill-proof.

**DSIC** TFMs. Dominant-strategy incentive-compatibility (DSIC) is one way to formalize the idea of a “good user experience (UX)” for TFMs. The condition asserts that every user has an “obviously optimal” bid, meaning a bid that, provided the BP follows the TFM’s allocation rule, is guaranteed to maximize the user’s utility (no matter what other users might be bidding). In the next definition, by a bidding strategy, we mean a function \( \sigma \) that maps a valuation to a recommended bid for a user with that valuation.

**Definition 2.10 (Dominant-Strategy Incentive-Compatibility (DSIC))** A TFM \((x, p, q)\) is dominant-strategy incentive-compatible (DSIC) if there is a bidding strategy \( \sigma \) such that, for every BP valuation \( v_{BP} \) and blockset \( B \), every user \( i \) with transaction \( t \), every valuation \( v_t \) for \( i \), and every choice of other users’ bids \( b_{\neq t} \),

\[
\sigma(v_t) \in \arg\max_{b_t} \left\{ u_t(b_t) \right\} ,
\]

where \( u_t \) is defined as in (2).

That is, bidding according to the recommendation of the bidding strategy \( \sigma \) is guaranteed to maximize a user’s utility.\(^6\) This is a strong property: a bidding strategy can depend only on what a user knows (i.e., its private valuation), while the right-hand side of (4) implicitly depends (through (2)) also on the bids of the other users and the BP’s preferences.

\(^6\) The term “DSIC” is often used to refer specifically to mechanisms that satisfy the condition in Definition 2.10 with the truthful bidding strategy, \( \sigma(v_t) = v_t \). Any mechanism that is DSIC in the sense of Definition 2.10 can be transformed into one in which truthful bidding is a dominant strategy, simply by enclosing the mechanism in an outer wrapper that accepts truthful bids, applies the assumed bidding strategy \( \sigma \) to each, and passes on the results to the given DSIC mechanism. (This trick is known as the “Revelation Principle”; see e.g. [46].)
Example 2.11 (EIP-1559) The EIP-1559 mechanism [12] is parameterized by a “base fee” \( r \), which for each transaction \( t \) (with size \( s_t \)) defines a reserve price of \( r \cdot s_t \). This mechanism charges each user its bid—\( p_t(B, b) = b_t \) for all \( t \in B \)—and the portion of this revenue generated by the base fee goes to holders rather than the BP. That is, the mechanism’s burning rule is \( q(B, b) = \sum_{t \in B} r \cdot s_t \). (We allow a BP to include transactions with \( b_t < r \cdot s_t \), but the BP must still burn the full amount \( r \cdot s_t \); see also Remark 2.13.)

Following [47], call the base fee \( r \) excessively low if the BP cannot fit all the transactions \( t \) satisfying \( b_t \geq r \cdot s_t \) into a single (feasible) block. When the base fee is not excessively low, the standard allocation rule for the EIP-1559 mechanism instructs the BP to include all transactions \( t \) for which \( b_t \geq r \cdot s_t \) (and to leave out any transactions \( t \) with \( b_t < r \cdot s_t \)). With a passive BP, this allocation rule is consonant with the payment and burning rules of the mechanism: In this case, including a transaction \( t \) in the block contributes precisely \( b_t - r \cdot s_t \) to the BPS, so a passive BP is motivated to include all and only the transactions for which this expression is nonnegative. With this allocation rule (and a base fee that is not excessively low), the TFM is also DSIC, with the bidding strategy \( \sigma \) defined by \( \sigma(v_t) = \min\{v_t, r \cdot s_t\} \).

With an active BP, however, the usual allocation rule above is no longer consonant with the payment and burning rules of the mechanism, even when the base fee is not excessively low: A BP might be motivated to include a transaction \( t \) with \( b_t < r \cdot s_t \), if the deficit can be compensated for with the BP’s own private value for including the transaction. Thus, this version of the EIP-1559 mechanism is not BPIC. The mechanism’s allocation rule can be redefined to restore consonance, by instructing the BP to choose the block that maximizes its BPS (rather than its revenue), but this robs the mechanism of its DSIC property: Intuitively, without knowing the BP’s valuation, a user cannot know whether to underbid (below its reserve price) to take advantage of a BP that might be motivated to include a transaction \( t \), or negative (otherwise). This implies that a passive BP cannot improve its BPS by deviating from the allocation rule’s recommendation. This TFM is also DSIC, under the same bidding strategy used in Example 2.11 or, alternatively, under the truthful bidding strategy.

The main result of Section 3 (Theorem 3.1) shows that the whack-a-mole between the DSIC and BPIC properties in Example 2.11 is not particular to the EIP-1559 mechanism: When BPs are active, no TFM that charges non-zero user fees can be both DSIC and BPIC.

Our final example shows that, with a passive BP, the DSIC and BPIC properties can be achieved simultaneously even without the assumption in Example 2.11 about the accuracy of a base fee.

Example 2.12 (Tipless Mechanism) In the tipless mechanism [47], the burning rule is the same as in Example 2.11 (i.e., \( q(B, b) = \sum_{t \in B} r \cdot s_t \)), while the payment rule changes from \( p_t(B, b) = b_t \) to \( p_t(B, b) = \min\{b_t, r \cdot s_t\} \) for \( t \in B \). The mechanism’s allocation rule instructs the BP to include only transactions \( t \) satisfying \( b_t \geq r \cdot s_t \) and, subject to this constraint and block feasibility, to maximize the total size of the included transactions. (Ties are broken according to some fixed ordering over feasible blocks.) The contribution of an included transaction to a BP’s revenue is either 0 (if \( b_t \geq r \cdot s_t \)) or negative (otherwise). This implies that a passive BP cannot improve its BPS by deviating from the allocation rule’s recommendation. This TFM is also DSIC, under the same bidding strategy used in Example 2.11 or, alternatively, under the truthful bidding strategy.

Off-chain agreements. For completeness, we briefly mention a third incentive-compatibility notion, which concerns a cartel of the BP and the users. Such cartels can in some cases coordinate off-chain to manipulate the intended behavior of a TFM. For example, one of the primary reasons that the EIP-1559 mechanism burns its base fee revenue is resilience to coordination of this type. (If that revenue were instead passed on to the BP, low-value users could collude with the BP to evade
the base fee, by overbidding on-chain to clear the base fee while accepting a rebate from the BP off-chain.) Informally, a TFM is OCA-proof if it is robust to collusion of this type. (“OCA” stands for “off-chain agreement”; see [47] for the precise definition.) OCA-proofness shaped the design of the EIP-1559 mechanism, and it and related notions are fundamental to the TFM impossibility results (with passive BPs) in [17, 16, 26]. OCA-proofness plays a limited role in this paper, as our impossibility results (Theorems 3.1 and 3.3) apply already to mechanisms that are merely DSIC and BPIC (and not necessarily OCA-proof).

**Remark 2.13 (OCAs and the Two Versions of the EIP-1559 and Tipless Mechanisms)**

In the versions of the EIP-1559 and tipless mechanisms described in Examples 2.11 and 2.12, a BP is free to include in a block any transaction it wants, whether or not the bid \( b_t \) submitted with the transaction is high enough to cover the required burn \( r \cdot s_t \). An alternative design would change the definition of transaction validity so that such transactions are ineligible for inclusion. There is effectively no difference between the two designs when BPs are passive: A rational such BP would never include a transaction with \( b_t < r \cdot s_t \), even were it free to do so. An active BP, however, will be motivated to include such a transaction if it has a sufficiently high private value for it.

Off-chain agreements render these second versions of the EIP-1559 and tipless mechanisms equivalent to those described in Examples 2.11 and 2.12 even with active BPs. The reason is similar to the reason why base fee revenue must be withheld from a BP: If users collude with a BP, they can always bid \( r \cdot s_t \) on-chain to ensure inclusion eligibility while accepting an off-chain rebate of \( r \cdot s_t - b_t \) from the BP.

### 2.5 Adding Competitive Searchers

Next we describe the changes to the basic model that are needed in Sections 4 and 5, in which we suppose that block proposers outsource the problem of value extraction to searchers.

**Searchers and bundles.** Searchers submit bundles to the blockchain protocol, where a *bundle* consists of a single user-submitted transaction \( t \) and any additional transactions needed to extract value from \( t \). We assume that there is a canonical way to extend a transaction with size \( s_t \) into a bundle, and denote by \( s'_t \) the size of the latter (with \( s'_t \geq s_t \)). For example, if \( t \) represents an AMM trade, the corresponding canonical bundle might include a subsequent backrunning trade. Just as users submit bids with their transactions, searchers submit bids with their bundles. We use the notation \( w \) for a generic bundle. When we wish to emphasize that a bundle involves the user-submitted transaction \( t \), we write it as \( t^i \), where \( i \) denotes the searcher that assembled the bundle.

A TFM now takes as input both transactions (with their user bids) and bundles (with their searcher bids), and its allocation, payment, and burning rules can depend on the bids of all users and all searchers. We assume that a TFM can distinguish between transactions and bundles, and can therefore treat them differently (e.g., the output of the payment rule can be defined differently for users and for searchers). Like users, searchers have private nonnegative valuations for bundle inclusion and quasi-linear utility functions. The DSIC condition is defined for searchers exactly as it is for users (Definition 2.10).

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7 For example, one way to interpret the difference between the EIP-1559 mechanism (Example 2.11) and the tipless mechanism (Example 2.12) is that, when the base fee is excessively low, the former mechanism gives up on DSIC (but retains OCA-proofness) while the latter gives up on OCA-proofness (but remains DSIC).
Blocks. Blocks can now include both transactions and bundles. Multiple searchers may submit bundles corresponding to the same transaction, but in a feasible block, a given transaction can be included (directly or as part of a bundle) at most once. The inclusion of a bundle that contains a transaction \( t \) necessarily implies the inclusion of \( t \) itself—in this sense, the space of feasible allocations is no longer downward-closed. Equivalently, a block now specifies a set of user-submitted transactions and, for each such transaction \( t \), the searcher (if any) responsible for the included bundle that contains \( t \). Users continue to have a private value \( v_t \) for inclusion (whether as part of a bundle or not).

Revised incentive-compatibility goals. Thus far, the addition of searchers strictly generalizes the model in Sections 2.1–2.4 and so our impossibility results (Theorems 3.1 and 3.3) for the basic model apply immediately to it as well.

But the whole point of accommodating a competitive ecosystem of searchers is for proposers (the entities that participate directly in the blockchain protocol) to outsource the specialized task of assembling high-value blocks to searchers. That is, searchers are meant to allow proposers to on the one hand act passively (by simply using the most valuable bundles submitted by searchers) and on the other hand earn almost all of the extractable value (with searchers competing the value of their bundles away to the proposer through the bidding process)\(^8\). Mathematically, with searchers, the idea is that what had been the private valuation \( v_{BP} \) of the (vertically integrated) BP in Section 2.1 is now distributed specifically across the searchers. This interpretation is particularly clear in the additive case—meaning the vertically integrated BP valuation \( v_{BP}(B) \) would have been \( \sum_{t \in B} \mu_t \), with \( \mu_t \) the value extractable from a transaction \( t \) and no interactions between different transactions—with every searcher that submits a bundle involving transaction \( t \) having a value of \( \mu_t \) for that bundle.\(^9\)

With this interpretation in mind, in the model with searchers, there will be three incentive-compatibility goals: (i) DSIC for users; (ii) DSIC for searchers; and (iii) BPIC for the proposer, assuming that the proposer is passive (i.e., with the all-zero valuation for blocks and with utility equal to the net revenue at the consensus layer, including any payments to it from searchers). In effect, this revised model shatters what had been a vertically integrated BP into a single proposer and a number of searchers, and what had been BPIC (with an active BP) now translates to DSIC for (active) searchers and BPIC for a passive proposer.\(^10\)

Welfare. With searchers, we redefine the welfare \([1]\) of a block \( B \) to reflect the private valuations of searchers and the fact that the proposer is assumed to have an all-zero valuation:

\[
W(B) := \sum_{t \in B_T} v_t + \sum_{w \in B_S} v_w,
\]

where \( B_T \) and \( B_S \) denote the transactions and bundles, respectively, in the block \( B \).

\(^8\)See [7] for a rigorous analysis of this idea.

\(^9\)Transactions that are well modeled as additive in this sense include trades on different AMMs, or once-per-block MEV opportunities such as top-of-block CEX-DEX arbitrage or liquidation opportunities (the latter two types modeled via a dummy transaction that has a user bid of zero but non-zero value for searchers).

\(^10\)The combination of (i)–(iii) can technically be achieved by using the tipless mechanism (Example 2.12) and always ignoring any searchers that might be present. Our interest in Section 4 will be the incentive-compatibility properties of a more interesting TFM that incorporates searchers in a way that resembles current practice; the goal in Section 5 is to design novel TFMs that, in addition to satisfying (i)–(iii) and unlike the searcher-excluding tipless mechanism, guarantee a constant fraction of the maximum-possible welfare.
3 An Impossibility Result for DSIC and BPIC Mechanisms

3.1 Can DSIC and BPIC Be Achieved Simultaneously?

The DSIC property (Definition 2.10) encodes the idea of a transaction fee mechanism with “good UX,” meaning that bidding is straightforward for users. Given the unilateral power of BPs in typical blockchain protocols, the BPIC property (Definition 2.9) would seem necessary, absent any additional assumptions, to have any faith that a TFM will be carried out by BPs as intended. One can imagine a long wish list of properties that we’d like a TFM to satisfy; can we at least achieve these two?

The tipless mechanism (Example 2.12) is an example of a TFM that is DSIC and BPIC in the special case of passive BPs. This TFM is also “non-trivial,” in the sense that users generally pay for the privilege of transaction inclusion. With active BPs, meanwhile, the DSIC and BPIC properties can technically be achieved simultaneously by the following “trivial” TFM: the payment rule \( p \) and burning rule \( q \) are identically zero, and the allocation rule \( x \) instructs the BP to choose the feasible block that maximizes its private value (breaking ties in a bid-independent way). This TFM is BPIC by construction, and it is DSIC because a user has no control over whether it is included in the chosen block (it’s either in the BP’s favorite block or it’s not) or its payment (which is always 0).

Thus, the refined version of the key question is:

Does there exist a non-trivial TFM that is DSIC and BPIC with active BPs?

3.2 Only Trivial Mechanisms Can Be DSIC and BPIC

The main result of this section is a negative answer to the preceding question. By the range of a bidding strategy \( \sigma \), we mean the set of bid vectors realized by nonnegative valuations: \( \{ \sigma(v) : v \geq 0 \} \), where \( \sigma(v) \) denotes the componentwise application of \( \sigma \).

**Theorem 3.1 (Impossibility of DSIC, BPIC, Non-Triviality)** If the TFM \((x, p, q)\) is DSIC with bidding strategy \( \sigma \) and BPIC with active block producers, then the payment rule \( p \) is identically zero on the range of \( \sigma \).

The proof of Theorem 3.1 will show that the result holds even if BPs are restricted to have nonnegative additive valuations and all known transactions can be included simultaneously into a single feasible block.

**Discussion.** The role of an impossibility result like Theorem 3.1 is to illuminate the most promising paths forward. From it, we learn that our options are (i) constrained; and (ii) already being actively explored by the blockchain research community. Specifically, with active BPs, to design a non-trivial TFM, we must choose from among three options:

1. Give up on “good UX,” at least as it is expressed by the DSIC property.

2. Give up on the BPIC property, presumably compensating with restrictions on block producer behavior (perhaps enforced using, e.g., trusted hardware [24] or cryptographic techniques [14]).
3. Expand the TFM design space, for example by incorporating order flow auctions (e.g., \cite{38}) or block producer competition (e.g., \cite{19}) to expose information about a BP’s private valuation to a TFM. We explore this idea further in Sections 4 and 5.

Proof of Theorem 3.1: Let \((x, p, q)\) be a TFM that is BPIC, and DSIC with the bidding strategy \(\sigma\). By the Revelation Principle (see footnote \cite{6}), we can assume that \(\sigma\) is the truthful bidding strategy (i.e., the identity function). Toward a contradiction, suppose there is a nonnegative additive BP valuation \(v_{BP}\), a BP blockset \(B\), a set of transactions with bids \(b\), and a transaction \(t^*\) such that \(p_{t^*}(B, b) > 0\), where \(B = x(b, v_{BP}, B)\). Because the pricing rule \(p\) is individually rational (see Section 2.3), we must have \(t^* \in B\). Because the TFM \((x, p, q)\) is BPIC, the block \(B\) must maximize the BP’s BPS over all blocks in its blockset \(B\).

We next define a modified BP valuation and a modified bid vector. First, let \(b^* = (0, b_{-t^*})\) denote the bid vector in which the original bid \(b_{t^*}\) for transaction \(t^*\) is dropped to 0 and all other bids are held fixed. Second, let \(P\) denote the sum of the bids of all known transactions (i.e., \(P = \sum_t b_t\)) and \(Q\) the burn that the TFM would require on the modified bid vector for the block \(B\) (i.e., \(Q = q(B, b^*)\)), and define a modified (but still additive) valuation \(\hat{v}_{BP}\) so that \(\hat{v}_{BP}(\{t\}) > v_{BP}(\{t\}) + P + Q\) for all \(t \in B\) and \(\hat{v}_{BP}(\{t\}) = 0\) for all \(t \notin B\).

The key claim is that the BPS-maximizing block \(x(b^*, \hat{v}_{BP}, B)\) for the modified valuation with the modified bid vector contains every transaction of \(B\), and in particular \(t^*\). Under this modified valuation and bid vector, the BPS of a block \(B' \in B\) can be written as

\[
\hat{v}_{BP}(B') = \sum_{t \in B'} p_t(B', b') - q(B', b').
\]

By the definition of \(\hat{v}_{BP}\), any transaction in \(B\) omitted from \(B'\) results in a loss of more than \(P + Q\) in the private valuation of the BP:

\[
\hat{v}_{BP}(B) > \hat{v}_{BP}(B') + P + Q
\]

for every feasible block \(B' \supseteq B\). Next, individual rationality of the payment rule \(p\) implies that the maximum revenue a BP can receive from including a transaction \(t\) is the attached bid \(b_t\), and thus the maximum revenue it receives from any block in \(B\) is at most \(P\). Because the payment rule \(p\) is nonnegative, we have

\[
\sum_{t \in B'} p_t(B', b') \leq \sum_{t \in B} p_t(B, b) + P
\]

for every \(B' \in B\). Finally, because the burning rule \(q\) is nonnegative,

\[
q(B, b') \leq q(B', b') + Q
\]

for every \(B' \in B\). Combining the inequalities (6)–(9) then implies that, with the modified valuation and bid vector, the BPS of the block \(B\) is strictly higher than that of every block that omits at least one of \(B'\)’s transactions:

\[
\frac{\hat{v}_{BP}(B') + P + Q}{\hat{v}_{BP}(B)} < \frac{\sum_{t \in B} p_t(B, b) - q(B, b)}{\hat{v}_{BP}(B')} < \frac{\sum_{t \in B'} p_t(B', b') - q(B', b')}{\hat{v}_{BP}(B')}
\]

16
for every $B' \not\supseteq B$. This completes the proof of the key claim.

The point of this claim is that, when the BP has valuation $\hat{v}_{BP}$ and blockset $B$ and the other transactions’ bids are $b_{-t^*}$, the transaction $t^*$ will be included in the BP’s chosen block $B' = x(b', \hat{v}_{BP}, B)$ even when its creator sets $b_{t^*} = 0$. Because the payment rule $p$ is individually rational, $p_{t^*}(b', B') = 0$. Because the user that created transaction $t^*$ can guarantee inclusion at price 0 with a bid of 0, any bid that leads to a positive price is automatically suboptimal for it. Because the TFM $(x, p, q)$ is DSIC with the truthful bidding strategy, $t^*$ must be included at a price of 0 also when its creator submits the original bid $b_{t^*}$; that is, if $\hat{B}$ denotes $x(b, \hat{v}_{BP}, B)$, then $t^* \in \hat{B}$ and $p_{t^*}(\hat{B}, b) = 0$.

We can complete the contradiction and the proof by arguing that $\hat{B} = B$. (This would imply that $p_{t^*}(B, b) = 0$, in direct contradiction of our initial assumption.) By definition, the block $B$ is a BPS-maximizing block for a BP with valuation $v_{BP}$ and blockset $B$ with transaction bids $b$, and it is the first such block with respect to some fixed ordering over $B$ (recall Definition $2.7(b)$). By construction of the modified valuation $\hat{v}_{BP}$, the block $B$ enjoys at least as large a private value increase $\hat{v}_{BP}(B) - v_{BP}(B)$ as any other block of $B$. Because the payment and burning rules of a TFM are independent of the BP valuation, holding the bids $b$ fixed, the block $B$ also enjoys at least as large a BPS increase as any other block of $B$. Thus, the BPS-maximizing blocks with respect to the modified valuation $\hat{v}_{BP}$ are a subset of those with respect to the original valuation $v_{BP}$, and this subset includes the block $B$. Because the allocation rule breaks ties consistently, $\hat{B} = x(b, \hat{v}_{BP}, B)$ must be the original block $B$. ■

Remark 3.2 (Variations of Theorem 3.1) Variations on the proof of Theorem 3.1 show that the same conclusion holds for:

(a) BPs that have a non-zero private value for only one block (a very special case of single-minded valuations). This version of the argument does not require the consistent tie-breaking assumption in Definition $2.7(b)$.

(b) Burning rules that need not be nonnegative (i.e., rules that can print money), provided that, for every bid vector $b$, there is a finite lower bound on the minimum-possible burn $\min_{\substack{B \in B}} q(B, b)$. (This would be the case if, for example, the blockset $B$ is finite.)

(c) Bid spaces and payment rules that need not be nonnegative (i.e., with negative bids and user rebates allowed, subject to individual rationality), provided there is a finite minimum bid $b_{min} \in (-\infty, 0]$ and that $p_{t}(B, b) = b_{min}$ whenever $t \in B$ with $b_{t} = b_{min}$. In this case, the argument shows that the payment rule $p$ must be identically equal to $b_{min}$ on the range of $\sigma$.

3.3 The Welfare Achieved by DSIC and BPIC Mechanisms

Theorem 3.1 shows that TFMs that are DSIC and BPIC must be “trivial,” in the sense that users are never charged for the privilege of transaction inclusion. The next result formalizes the intuitive consequence that such TFMs may, if both users and the BP follow their recommended actions, produce blocks with welfare arbitrarily worse than the maximum possible. (Recall that the welfare $W(B)$ of a block $B$ is defined in expression (1) in Section 2.2.) That is, no approximately welfare-maximizing TFM can be both DSIC and BPIC with active BPs. This result is not entirely trivial because the conclusion of Theorem 3.1 imposes no restrictions on the burning rule of a TFM.
Theorem 3.3 (Impossibility of DSIC, BPIC, and Non-Trivial Welfare Guarantees) Let \((x, p, q)\) denote a TFM that is BPIC and DSIC with bidding strategy \(\sigma\). For every approximation factor \(\rho > 0\), there exists a BP valuation \(v_{BP}\), BP blockset \(B\), block \(B^* \in B\), and transactions with corresponding user valuations \(v\) such that

\[
W(B) \leq \rho \cdot W(B^*),
\]

where \(B = x(\sigma(v), v_{BP}, B)\).

Proof: Let \((x, p, q)\) denote a TFM that is DSIC and BPIC. By Theorem 3.1, the payment rule \(p\) is identically zero on the range of its recommended bidding strategy \(\sigma\). We assume that (appealing to DSIC) users always follow this bidding strategy \(\sigma\) and that (appealing to BPIC) the BP always chooses the block recommended by the allocation rule \(x\). By the Revelation Principle (see footnote 6), we can assume that \(\sigma\) is the identity function.

Suppose there are two known transactions, \(y\) and \(z\), with arbitrary positive user valuations \(v_y\) and \(v_z\). Suppose the BP blockset \(B\) comprises three feasible blocks, \(B_0 = \emptyset\), \(B_y = \{y\}\), and \(B_z = \{z\}\). Set \(v_{BP}(B_0) = v_{BP}(B_y) = 0\) and

\[
v_{BP}(B_z) = q(B_z, v) + \epsilon
\]

for some small \(\epsilon > 0\). Then, because the burning rule \(q\) is nonnegative and the payment rule \(p\) is identically zero, the BP will choose the block \(B_z\) (i.e., \(x(v, v_{BP}, B) = B_z\)).

To complete the proof, we range over all valuation vectors of the form \(v' = (v'_y, v_z)\) and treat separately three (non-exclusive but exhaustive) cases:

- **(C1)** Every choice of \(v'_y\) leads the BP to choose \(B_z\) (i.e., \(x(v', v_{BP}, B) = B_z\) for all \(v'_y\)).
- **(C2)** Some choice of \(v'_y\) leads the BP to choose \(B_y\) (i.e., \(x(v', v_{BP}, B) = B_y\)).
- **(C3)** Some choice of \(v'_y\) leads the BP to choose the empty block (i.e., \(x(v', v_{BP}, B) = B_0\)).

In case (C1), because the BP always, no matter the value of \(v'_y\), chooses the block \(B_z\) (with welfare \(v_z + q(B_z, v) + \epsilon\)) over the block \(B_y\) (with welfare \(v'_y\)), letting \(v'_y\) tend to infinity proves the desired result (with \(B = B_z\) and \(B^* = B_y\)).

Case (C2) cannot occur, for if it did, the creator of transaction \(y\) would prefer to misreport its valuation (as \(v'_y\)) when its true valuation is \(v_y\), contradicting the assumption that the TFM \((x, p, q)\) is DSIC with the truthful bidding strategy. (Because \(p\) is identically 0 and \(v_y > 0\), the creator of \(y\) always strictly prefers inclusion to exclusion.)

Finally, in case (C3), the result follows immediately from the facts that \(W(B_0)\) is zero while \(W(B_y)\) and \(W(B_z)\) are positive. ■

Remark 3.4 (Generalizations of Theorem 3.3) The proof of Theorem 3.3 shows that the result holds already with BPBs that have additive or single-minded valuations. (As discussed in Remark 3.2, Theorem 3.1 holds in both these cases, and the BP valuation \(v_{BP}\) used in the proof of Theorem 3.3 is both additive and single-minded). A slight variation of the proof shows that the result holds more generally for DSIC and BPIC TFM that use a not-always-nonnegative burning rule, under the same condition as in Remark 3.2(b).
4 Transaction Fee Mechanisms with Searchers

4.1 Incorporating Searchers

The impossibility results in Section 3 are consistent with practice, in the sense that modern attempts to mitigate the negative consequence of MEV through economic mechanisms generally lie outside the basic design space of TFMs introduced in Sections 2.1–2.4. The most popular such mechanisms distribute the task of block production across multiple parties; in this section and the next, we adopt the model described in Section 2.5, which captures some of this complexity through the addition of searchers that can submit bundles (of a user-submitted transaction together with the searcher’s value-extracting transactions) to a TFM. Recall from Section 2.5 that, in this model, what had been the private valuation $v_{BP}$ of a vertically integrated BP is effectively distributed across a set of searchers, with the block proposer, having outsourced the task of value extraction, then acting passively to maximize its revenue (including the payments from searchers for included bundles). The winning bid of a searcher can be interpreted as an “MEV oracle” that provides a TFM with an estimate of the value that can be extracted from the bundled transaction. In this sense, the TFM design space with searchers is richer than the basic model with users only, and there is hope that a TFM can take advantage of such estimates to define payments for user-submitted transactions in a DSIC-respecting way (e.g., with searchers’ bids leading in some cases to user refunds). Indeed, we’ll see that this expanded design space allows for positive results that would be impossible in the basic model with active BPs.

In this section, we propose an abstraction of how searchers have traditionally been incorporated into the block production process, inspired specifically by mev-geth (see Section 2.5), and study the incentive-compatibility properties of the resulting mechanism. Section 5 explores the TFM design space with searchers more generally, with a focus on welfare guarantees.

4.2 The s-Tipless Mechanism

We next introduce the Searcher Augmented Tipless Mechanism (s-tipless mechanism). Like the EIP-1559 and tipless mechanisms (Examples 2.11 and 2.12), it has a fixed base fee $r$ that is charged per unit size. Intuitively, for each user-submitted transaction $t$, the mechanism runs a first-price auction among the interested searchers; such an auction is often referred to as an “order-flow auction.” (Thus, the mechanism does not attempt to be DSIC for searchers.) If the winning bid $b_w$ in this auction is high enough to pay the base fee charges (i.e., $b_w \geq r \cdot s'_t$, where $s'_t$ is the size of a bundle that contains $t$), then $w$’s bundle is included in the block and $w$ pays its bid (while the user that submitted $t$ pays nothing). If the winning searcher bid is less than $r \cdot s'_t$ then, if the user that submitted $t$ bids at least the relevant base fee charges (i.e., $b_t \geq r \cdot s_t$), the transaction $t$ is included in the block and the submitting user pays $r \cdot s_t$. In either case, all base fee revenues ($r \cdot s_t$ or $r \cdot s'_t$) are burned. (The block proposer may still collect revenue from the first-price auction among searchers if the winning bid exceeds $r \cdot s'_t$.) In effect, searchers can cover base fee charges for a user if their transaction is sufficiently valuable to them.

**Definition 4.1 (Searcher-Augmented Tipless Mechanism (s-tipless mechanism))** Fix a base fee $r \geq 0$:

(a) **Allocation rule:** A transaction should be included if its bid clears its base fee payment, or it is contained in a bundle with a bid that clears the bundle’s base fee payment. If multiple
bundles for a transaction have bids that clear the base fee payment, the bundle with the highest bid is included (breaking ties arbitrarily). Formally, for each \( t \in T \), let \( S_t \) denote the submitted bundles that contain \( t \), \( w \) a generic such bundle, and \( t^* = \arg\max_{w \in S_t} \{b_w\} \). Define

\[
S^* = \{ t^* : t \in T, b_{t^*} \geq r \cdot s_t \} \quad \text{and} \quad T^* = \{ t \in T : b_t \geq r \cdot s_t \lor S_t \cap S^* \neq \emptyset \},
\]

and the allocation rule by

\[
x(b, B) = T^* \cup S^*.
\]

(b) **Payment rule:** For all transactions \( t \) in a block \( B \):

\[
p_t(B, b) = \begin{cases} 
0 & \text{if } S_t \cap B \neq \emptyset \\
 r \cdot s_t & \text{otherwise.}
\end{cases}
\]

For all bundles \( w \) in a block \( B \):

\[
p_w(B, b) = b_w.
\]

(c) **Burning rule:** For a block \( B \) with transactions \( B_T \) and bundles \( B_S \):

\[
q(B, b) = \sum_{t \in B_T} r \cdot s_t + \sum_{w \in B_S} r \cdot (s'_t - s_t).
\]

In Definition 4.1 and Theorem 4.2 below, we assume for simplicity that the base fee \( r \) is large enough that \( T^* \cup S^* \in B \), meaning there is sufficient room in a block for all of the transactions that the mechanism would like to include (i.e., all transactions for which either the user or some searcher is willing to cover the relevant base fee charges). In practice, a la the EIP-1559 mechanism, the base fee \( r \) would generally be adjusted by local search so that this property typically holds. Definition 4.1 and Theorem 4.2 can be extended to the general case (with contention between sufficiently high-bidding transactions and bundles) by redefining the allocation rule to maximize the total proposer revenue (i.e., \( \sum_{t \in B_S} (b_{t^*} - r \cdot s'_t) \)) over \( B \in \mathcal{B} \), breaking ties in a consistent way.

**Theorem 4.2 (Properties of the s-Tipless Mechanism)** The s-tipless mechanism is DSIC for users and BPIC.

Informally, to see that the mechanism is DSIC for users, note that if a transaction has a bundle included for it, then it always pays 0 regardless of what it bids, trivially giving the user a dominant bidding strategy. Otherwise, the user faces a fixed price for inclusion and hence has a dominant strategy to bid above that price if and only if their value exceeds it. To see that the mechanism is BPIC, note that the only revenue the BP gets is from searcher bids that exceed the base fee charges. Standalone transactions have no net effect on the BP’s BPS. Thus any allocation rule that includes the highest-bidding searchers that clear their base fee charges along with the transactions that clear their base fee charges is consonant. Furthermore, because the searchers’ payments depend only on their own bids, the BP has no way to increase their BPS through the insertion of shill bids. A formal proof of Theorem 4.2 follows.

---

11 We subtract \( s_t \) for every bundle \( w \in B_s \) so as not to double-count \( s_t \) both as part of a bundle and as a standalone transaction.
Proof of Theorem 4.2: We first show that the s-tipless mechanism is DSIC for users. Because the setting is single-parameter, it suffices to show that the allocation rule is monotone in users’ bids and that users pay their minimal bids for inclusion. Fix a user with transaction \( t \). If there is a bundle containing \( t \) with bid at least \( r \cdot s'_t \), then \( t \) is always included and the user is charged 0 (the minimal bid for inclusion). Otherwise, \( t \) is included if and only if \( b_t \geq r \cdot s_t \), in which case the user pays \( r \cdot s_t \) (again, the minimal bid for inclusion).

For BPIC, we first note that the mechanism is shill-proof (Definition 2.8): adding fake transactions with shill bids to a block has no effect on the payments or burns associated with the other transactions in the block, and thus only causes the BP to suffer a larger burn. Next, note that the BP is disincentivized to include any transactions or bundles outside of \( S^* \cup T^* \) (i.e., with bids that do not cover the corresponding base fee charges); deleting such a transaction or bundle from a block does not affect the payments or burns associated with the other transactions, and therefore results in a new block with a strictly higher BPS. Restricting attention accordingly to the transactions and bundles in \( S^* \cup T^* \), because the payments and burns of the included transactions and bundles are independent, it suffices to consider each transaction \( t \) in isolation. The BP can: (i) include \( t \) directly; (ii) include \( t \) as part of the highest-bidding corresponding bundle \( w \) (with bid \( b_w \)); or (iii) exclude \( t \). The BP’s revenue from \( t \) in case (iii) is 0. Its revenue from \( t \) in case (i) is \( \min\{0, b_t - r \cdot s_t\} \). Its revenue from \( w \) in case (ii) is \( b_w - r \cdot s'_t \). Thus, the BP’s revenue from \( t \) is maximized by following the recommendation of the allocation rule: including \( w \) if the revenue in case (ii) is nonnegative, and otherwise including \( t \) directly for zero revenue. (Because \( t \in S^* \cup T^* \), \( b_w < r \cdot s'_t \) implies that \( b_t \geq r \cdot s_t \) and hence \( \min\{0, b_t - r \cdot s_t\} = 0 \).) ■

5 Welfare Guarantees

This section further investigates transaction fee mechanism design in the presence of searchers, as in the model in Section 2.5 with a focus on welfare guarantees.

5.1 What Do We Want from a TFM?

Starting from a blank page, we naturally want to design a mechanism that scores well with respect to all the criteria we have considered thus far:

(P1) DSIC for users;

(P2) DSIC for (active) searchers;

(P3) BPIC (with a passive block proposer);

(P4) good welfare guarantees.

Without searchers and with an active BP, Theorem 3.3 shows that the combination of (P1), (P3), and (P4) is unachievable. We also noted in passing (footnote 10) that the tipless mechanism (Example 2.12), modified to always ignore searchers, satisfies (P1)–(P3). (Such a mechanism can obviously lead to a highly welfare-suboptimal outcome when the valuations of searchers are significantly bigger than those of the users.)

Given the welfare-maximization goal (P4), one obvious starting point is the Vickrey-Clarke-Groves (VCG) mechanism, which in this context would accept bids from all users and searchers,
output a feasible block that maximizes the social welfare \((5)\) (taking users’ and searchers’ bids at face value), and charge each included user or searcher its externality (i.e., the loss in welfare its bid causes to others). As always, the VCG mechanism is DSIC (in this case, for both users and searchers) and maximizes the social welfare at its dominant-strategy equilibrium. It does not, however, satisfy property (P3). For example, even with only one user-submitted transaction and a number of corresponding searchers (i.e., a second-price auction), the block proposer is generally incentivized to masquerade as a searcher and insert a shill bid (just below the highest searcher bid) to increase its revenue. (A similar problem arises if the \(s\)-tipless mechanism in Section 4 is defined with second-price rather than first-price searcher auctions.)

One easy way to turn the VCG mechanism—or really, any TFM with a passive block proposer—into a BPIC mechanism is to always burn all the payments made by users and searchers. The block proposer would then be indifferent over blocks and willing to carry out an arbitrary allocation rule. An extension of this idea that attempts to trade welfare for a non-zero amount of BP revenue would be to use bidder-specific reserve prices (like \(r \cdot s_t\) and \(r \cdot s'_t\) in the \(s\)-tipless mechanism), the revenue from which is not burned.

Summarizing, the VCG mechanism with all payments burned satisfies all of (P1)–(P4), and in particular shows that the addition of searchers allows TFMs to circumvent the impossibility result in Theorem 3.3. Should we declare victory?

### 5.2 Sybil-Proof Mechanisms

In a permissionless blockchain protocol like Bitcoin or Ethereum, it is easy to generate multiple identities in an undetectable way. For example, a user can easily participate as a “fake searcher” in a TFM if it so chooses. This challenge of “sybils,” especially in tandem with the non-downward-closed nature of the set of feasible blocks (with inclusion of a bundle implying inclusion of the corresponding transaction), renders the VCG mechanism extremely easy to manipulate (despite being DSIC for users and searchers separately).

For example, consider an instance with a block size of \(k\) in which all transactions and bundles have unit size and with one searcher per transaction, i.e., \(s_t = s'_t = 1\) and \(S_t = \{t^*\}\) for all \(t \in T\). In this case, the VCG mechanism will include the transactions and bundles corresponding to the \(k\) highest values of \(b_t + b_{t^*}\). Let the \((k + 1)\)th-highest of these values be \(r\). The included user and searcher for transaction \(t\) would then pay \(\max\{r - b_{t^*}, 0\}\) and \(\max\{r - b_t, 0\}\), respectively. Thus, if both \(b_t \geq r\) and \(b_{t^*} \geq r\), neither user nor searcher pays anything at all. There is then an obvious incentive for a user to submit an arbitrarily high bid while simultaneously masquerading as a searcher and submitting an arbitrarily high searcher bid. Every user with a non-zero valuation for inclusion is incentivized to do this. And if users engage in such manipulations, the welfare of the outcome of the mechanism could be very far from optimal. The ease and costs of such manipulations motivate seeking out TFMs that are, among other properties, “sybil-proof” in some sense.

Our definition of sybil-proofness (for users and searchers) mirrors our definition of BPIC, in that it asserts that the party in question cannot increase their utility through the submission of shill bids.\(^\text{12}\)

\(^{12}\)A mechanism with any non-zero reserve prices cannot offer any worst-case approximate welfare guarantees: for all the mechanism knows, only one participant has a non-zero valuation, which is just below the mechanism’s non-zero reserve price for that participant. We leave a Bayesian analysis (e.g., with the choice of reserve prices informed by historical bidding data) of the revenue-welfare trade-offs of such mechanisms to future work.
bids. In the following definition, $\bar{u}_i(b')$ denotes the utility of a user or searcher $i$ when submitting the bid vector $b' = (b'_1, \ldots, b'_m)$ for the transactions and bundles $(y_1, \ldots, y_m)$ under $m$ different identities (with respect to fixed bids $b_{-i}$ for the other participants). We assume that $i$ has one real transaction or bundle $y^*$, for which it has value $v_i$; any number of the $y_j$'s may equal $y^*$. We assume that $i$ may use any identities that are distinct from the identities used by other agents. Agent $i$'s utility $\bar{u}_i(b')$ is then $v_i$ (if at least one $y_i$ with $y_i = y^*$ is included in the block chosen by the TFM) or 0 (otherwise), minus the combined payment the TFM charges to its $m$ identities.

**Definition 5.1 (Sybil-Proofness)** A transaction fee mechanism $(x, p, q)$ is sybil-proof if for every user or searcher $i$, every set of bids $b_{-i}$ for the other agents, and every vector of bids $b'$, there exists some bid $b_i$ such that $u_i(b_i) \geq \bar{u}_i(b')$, where $u_i(\cdot)$ denotes the utility of a single bid under the agent’s true identity for its real transaction or bundle, and $\bar{u}_i(\cdot)$ denotes the agent’s utility when submitting multiple bids for transactions and/or bundles under multiple identities.

Intuitively, this definition asserts that a user or searcher should never earn more utility from submitting bids under multiple identities than they could have through a single bid for their transaction or bundle under their true identity.

We now augment our previous desiderata with:

(P5) sybil-proof.

Next we provide a TFM that satisfies the full set (P1)–(P5) of desired properties.

## 5.3 The Searcher-Augmented Knapsack Auction

We assume in this section that block feasibility depends only on the validity and the total size of the included transactions (which should be at most the block size $k$). We consider a mechanism that chooses which transactions and bundles to include based on their bid-to-size ratios. For ease of exposition, we assume that these ratios are distinct. (This assumption can be removed through standard lexicographic tie-breaking.) The mechanism finds a threshold ratio such that all transactions and bundles that have bid-to-size ratios above this threshold can fit into a block. This ratio is then used as a per-size price charged to included transactions and bundles. Similarly to the s-tipless mechanism, an included bundle pays all the costs for its corresponding transaction. For an included bundle, in the case that the second-highest searcher bid for the transaction is greater than the threshold payment, the winning searcher pays this second-highest bid instead. Finally, the burning rule is set to be the sum of users’ and searchers’ payments so that the block proposer always receives zero BPS.

**Definition 5.2 (Searcher-Augmented Knapsack Auction (SAKA))**

(a) **Allocation rule:** Recall that $t^*$ denotes the bundle with the highest bid for transaction $t$. For a given $\mu$, let

$$S^\mu = \{t^* : t \in T, b_{t^*}/s_{t^*} \geq \mu\} \text{ and } T^\mu = \{t \in T : b_t/s_t \geq \mu \lor S_t \cap S^\mu \neq \emptyset\}.$$ 

Then let $B^\mu = T^\mu \cup S^\mu$ be the block consisting of all transactions and bundles that have a bid-to-size ratio greater than $\mu$\footnote{Subject to the usual constraint that each transaction is included (by itself or as part of a bundle) at most once.}.
Define $\mu^* := \inf \{ \mu : \sum_{t \in B_T^u} s_t + \sum_{t' \in B_S^u} (s'_t - s_t) \leq k \}$, where $B_T^u$ and $B_S^u$ denote the transactions and bundles, respectively, in the block $B^u$. Then,

$$x(b, B) = B^{u*}.$$ 

(b) **Payment rule:** Define $b'_t := \max_{t' \in S^t, t' \neq t} \{ b_{t'} \}$ as the second-highest bundle bid for transaction $t$. (If there is no such bid, interpret $b'_t$ as 0.) For $t \in B_T$:

$$p_t(B, b) = \begin{cases} 0 & \text{if } S_t \cap B = \emptyset \\ \mu^* \cdot s_t & \text{otherwise.} \end{cases}$$

For $t^i \in B_S$:

$$p_{t^i}(B, b) = \max \{ \mu^* \cdot s'_t, b_{t' \notin B} \}.$$ 

(c) **Burning rule:**

$$q(B, b) = \sum_{t \in B_T} p_t(B, b) + \sum_{w \in B_S} p_w(B, b).$$

5.4 Analysis

We consider the incentive-compatibility properties of the SAKA mechanism in Theorem 5.3 and its welfare guarantee in Theorem 5.5. We conclude with Theorem 5.7, which shows that the welfare guarantee in Theorem 5.5 is near-optimal among TFMs that satisfy properties (P1)–(P5).

**Theorem 5.3 (Incentive-Compatibility Properties of the SAKA Mechanism)** The Searcher-Augmented Knapsack Auction (SAKA) mechanism is DSIC for both users and searchers, BPIC, and sybil-proof.

We give the main ideas of the proof here and the full proof below. The BPIC property follows immediately from the burning rule. To see that the mechanism is DSIC for users, we can focus on a transaction that is not included as part of a bundle (otherwise, the user pays 0). The allocation rule is monotone because once a user’s bid clears $\mu^* \cdot s_t$ they will always be included. Furthermore, $\mu^* \cdot s_t$ is the minimal amount $t$ can bid and be included; being included at any lower bid would contradict the definition of $\mu^*$. The mechanism is DSIC for searchers for similar reasons, with the addition of needing to pay at least the second-highest searcher bid to still be included. Sybil-proofness follows from the fact that $\mu^*$ is weakly increasing in the number of bids, and thus users and searchers have no way to decrease their payment by bidding on fake transactions.

**Proof of Theorem 5.3** The BPIC property is immediate (due to the choice of burning rule). Next we show that the mechanism is DSIC for users. In the case that a bundle $t^*$ is included, $t$ pays 0 and is included regardless of what it bids, so bidding truthfully is trivially an optimal strategy. So, assume that no included bundle includes transaction $t$. Fixing the other bids $b_{-t}$, it suffices to show that the allocation rule is monotone in $b_t$ and that $t$ pays its minimal price for inclusion.

To see that the allocation rule is monotone in $b_t$, note that once $t$ is included, increasing $b_t$ further has no effect on $\mu^*$. Thus, because $t$ is included as long as $b_t \geq \mu^* \cdot s_t$, $t$ will continue to be included for all higher bids. To see that $\mu^* \cdot s_t$ is the minimal price for inclusion for $t$, let $t$ drop its bid to $b_t' < \mu^* \cdot s_t$ and let $\tilde{\mu}$ be the equivalent of $\mu^*$ for the bid vector $b = (b_t', b_{-t})$. We claim
that \( \hat{b}_t < \hat{\mu} \cdot s_t \). Otherwise, we would have \( \sum_{t \in B^\hat{\mu}_S} s_t + \sum_{t' \in B^\hat{\mu}_S} (s'_t - s_t) \leq k \) under bid vector \( \hat{b} \), with the same inequality holding under the bid vector \( b \) (as all the bids in \( b_{-t} \) are identical in both cases); given that \( \hat{\mu} \leq \hat{b}_t / s_t < \mu^* \), this would contradict the definition of \( \mu^* \). The argument that the mechanism is DSIC for searchers mirrors that for users, with the addition that, because only highest-bidding searchers are included, a searcher’s minimal price for inclusion is the maximum of \( \mu^* \cdot s'_t \) and \( b'_t \).

Finally, we show that the SAKA mechanism is sybil-proof. Given that it is DSIC for users and searchers, it suffices to consider whether any agent can use sybils to increase their utility compared to bidding their true value. Note that \( \mu^* \) and \( b'_t \) for all \( t \in T \) can only increase as additional bids are inserted. Thus, a searcher can never increase their utility through the addition of fake bids, and a user can never increase their utility through the addition of fake transaction bids or fake searcher bids for other transactions. It remains to show that a user cannot increase their utility by adding fake searcher bids for their own transaction. This follows from the fact that, given that the size \( s'_t \) of a bundled transaction is at least the size \( s_t \) of the transaction itself, the winning searcher always pays at least as much for its bundle (\( \max\{\mu^* \cdot s'_t, b'_t\} \)) as the corresponding user would for its transaction (\( \mu^* \cdot s_t \)).

**Remark 5.4 (Deferred Acceptance Implementation)** The SAKA auction can be implemented as a deferred acceptance mechanism and is therefore also robust to certain forms of collusion between users and searchers (formally, the mechanism is weakly groupstrategy-proof).

Because the allocation rule of the SAKA mechanism differs from that of the VCG mechanism, we can immediately deduce that the SAKA mechanism does not always output the maximum-welfare solution. We parameterize the mechanism’s welfare guarantee by the maximum fraction \( \gamma \) of a block’s capacity that is consumed by a single transaction or bundle. (In many blockchain protocols, \( \gamma \) is typically 2% or less.)

**Theorem 5.5 (Welfare Guarantee of the SAKA Mechanism)** Assuming truthful bids by users and searchers, the social welfare of the block \( B^\mu_\ast \) output by the SAKA mechanism is at least \( (1 - \gamma) / 2 \) times that of a welfare-maximizing block \( B^\ast \), where \( \gamma \) denotes the fraction of a block that can be consumed by a single transaction or bundle.

The SAKA mechanism effectively implements as its allocation rule a standard greedy approximation algorithm for the knapsack problem, with the twist that it scores bundles’ densities according to \( v_t / s'_t \) rather than their true densities (i.e., \( (v_t + v_t^*) / s'_t \)). Intuitively, this twist cannot cause more than a factor-2 loss in social welfare because any bundle from the optimal solution that is not included is replaced by a transaction or bundle with density at least half as large. We can then use the fact that the greedy algorithm fills up at least a \( 1 - \gamma \) fraction of the block in order to get the final bound.

**Proof of Theorem 5.5:** Let \( B^\ast \) denote a welfare-maximizing block. Assume that \( \mu^* > 0 \), as otherwise the SAKA mechanism includes all transactions and bundles and therefore achieves the maximum-possible social welfare. Let \( \tilde{B} \) denote the transactions and bundles that are included in both \( B^\ast \) and \( B^\mu_\ast \) but excluding transactions that have an associated bundle included in \( B^\ast \) but not in \( B^\mu_\ast \):

\[
\tilde{B} = (B^\ast \cap B^\mu_\ast) \setminus \{t \in B^\mu_\ast : t^* \in B^\ast \setminus B^\mu_\ast\}.
\]
Define
\[ Q^* = \left\{ t^* \in B^* \setminus B^\mu : \frac{v_{t^*}}{s_t} < \frac{1}{2} \frac{v_t + v_{t^*}}{s_t}, \frac{v_t}{s_t} \geq \mu^* \right\}. \]

Denote by \( Q \) the transactions that correspond to bundles of \( Q^* \). Because every such transaction satisfies \( v_t \geq \mu^* s_t \), every such transaction belongs to \( B^\mu \). Both \( Q \) and \( Q^* \) are, by definition, disjoint from \( \tilde{B} \). Let \( R \) and \( R^* \) denote the sets of remaining transactions and bundles in \( B^\mu \setminus (\tilde{B} \cup Q) \) and \( B^* \setminus (\tilde{B} \cup Q^*) \), respectively. From these definitions we have \( W(B^\mu) = W(\tilde{B}) + W(Q) + W(R) \) and \( W(B^*) = W(\tilde{B}) + W(Q^*) + W(R^*) \).

Denote the total size of all the transactions and bundles in a set \( X \) by \( s_X \). We next note that \( W(Q) > \frac{1}{2} W(Q^*) \) and that \( s_Q \leq s_{Q^*} \). Specifically, for every bundle \( t^* \in Q^* \), we have that \( v_{t^*}/s_t < (v_t + v_{t^*})/2s_t \) and hence \( v_{t^*} < v_t \) and \( v_t > (v_t + v_{t^*})/2 \). Also, because each transaction \( t \) of \( Q \) corresponds to a bundle \( t^* \) of \( Q^* \) and \( s_t \leq s_t^* \), \( s_Q \leq s_{Q^*} \).

To bound \( W(R^*) \) from above, we use the following lemma.

**Lemma 5.6** For every transaction \( t \in R^* \), \( \frac{v_t}{s_t} < \mu^* \); for every bundle \( t^* \in R^* \), \( \frac{v_{t^*} + v_t}{s_{t^*}} < 2\mu^* \).

Applying Lemma 5.6 to each transaction and bundle in \( R^* \) and summing up the resulting inequalities then shows that \( W(R^*) \leq 2\mu^* s_{R^*} \).

**Proof of Lemma 5.6.** If a transaction \( t \) belongs to \( R^* \), it does not belong to \( \tilde{B} \) and hence does not belong to \( B^\mu \), implying that \( v_t/s_t < \mu^* \). Now consider a bundle \( t^* \in R^* \). Because \( t^* \notin \tilde{B} \), \( v_{t^*}/s_t < \mu^* \). If \( v_t \leq v_{t^*} \), then \( (v_t + v_{t^*})/s_t \leq 2v_{t^*}/s_t < 2\mu^* \), as required. So suppose \( v_t > v_{t^*} \). In this case, \( t^* \notin Q^* \) implies that \( v_t/s_t < \mu^* \) (and therefore \( v_t/s_t < \mu^* \)); again, \( (v_t + v_{t^*})/s_t < 2\mu^* \), as required.

Returning to the proof of Theorem 5.5 we next claim that \( s_{B^\mu^*} > (1 - \gamma)k \). By our choice of \( \mu^* \), for every \( \mu < \mu^* \), \( B^\mu \) has size greater than \( k \). Because all transactions and bundles have distinct densities, for \( \mu \) sufficiently close to \( \mu^* \), \( B^\mu \) has exactly one more transaction or bundle than \( B^\mu^* \). Because the size of every transaction or bundle is at most \( \gamma \cdot k \), \( s_{B^\mu^*} \geq s_{B^\mu} - \gamma k > (1 - \gamma)k \).

Writing \( \mu_Q = W(Q)/s_Q \geq \mu^* \) and putting everything together, we can derive the desired welfare guarantee:

\[
\frac{W(B^\mu)}{W(B^*)} = \frac{W(\tilde{B}) + W(Q) + W(R)}{W(\tilde{B}) + W(Q^*) + W(R^*)} > \frac{\mu^* s_{\tilde{B}} + \mu_Q s_Q + \mu^* s_R}{\mu^* s_{\tilde{B}} + 2\mu_Q s_Q + 2\mu^* s_R} \quad (10)
\]

\[
> \frac{\mu^* s_{\tilde{B}} + \mu_Q s_Q + \mu^*(1 - \gamma)k - s_{\tilde{B}} - s_{Q}}{\mu^* s_{\tilde{B}} + 2\mu_Q s_Q + 2\mu^* (k - s_{\tilde{B}} - s_{Q})} \quad (11)
\]

\[
= \frac{s_Q (\mu_Q - \mu^*) + (1 - \gamma)\mu^* k}{2s_Q (\mu_Q - \mu^*) + 2\mu^* k - \mu^* s_{\tilde{B}}}
\]

\[
\geq \frac{1 - \gamma}{2},
\]

where in (10) we use the facts that every bundle and transaction of \( \tilde{B} \subseteq B^\mu \) has value-to-size ratio at least \( \mu^* \), \( W(Q^*) < 2W(Q) \), and \( W(R^*) < 2\mu^* s_{R^*} \); and in (11), we use that \( s_{B^\mu^*} > (1 - \gamma)k \), \( s_{B^\mu} \leq k \), and \( s_Q \leq s_{Q^*} \).
Our final result shows that, modulo the factor of $1 - \gamma$—which, as discussed above, is typically close to 1 in our context—the welfare approximation guarantee in Theorem 5.5 is optimal among deterministic mechanisms that are both DSIC (for users and searchers) and sybil-proof in the sense of Definition 5.4.

**Theorem 5.7 (Limitations on the Welfare of DSIC and Sybil-Proof Mechanisms)**

No deterministic TFM that is DSIC for users and searchers and sybil-proof can achieve better than a $1/2$-approximation to the optimal social welfare, even when transaction sizes are a negligible fraction of the block size.

**Proof:** We first give a proof that uses large transactions, and then extend the argument to hold for arbitrarily small transactions. Let $M = (x, p, q)$ be a TFM that is both DSIC (for users and searchers) and sybil-proof. Let $\epsilon > 0$ be arbitrary. We show that $M$ cannot achieve at least a $\frac{1}{2} + \epsilon$ fraction of the maximum-possible welfare in all of three instances $I_1, I_2,$ and $I_3$. In all instances, the block size $k$ is 1, the transaction set is $T = \{t_1, t_2\}$, and the bundle set is $S = \{t'_1, t'_2\}$. Both transactions have unbundled and bundled sizes of 1 (i.e., $s_{t_1} = s'_{t_1} = s_{t_2} = 1$). The three instances will differ only in the valuations of the transactions and bundle. Appealing to DSIC and the Revelation Principle, when analyzing the welfare achieved by $M$, we can assume that each user and searcher bids their valuation. We use $B^*$ to denote the (feasible) block $\{t_1, t'_1\}$, which will be the maximum-welfare block in each of the three instances.

Define $I_1$ by $v_{t_1} = 1 - \epsilon/4$, $v_{t'_1} = 1 - \epsilon/4$, and $v_{t_2} = 1$; denote this valuation vector by $v^1$. Note that $M$ must choose $B^*$ to achieve a $(1/2 + \epsilon)$-approximation of the maximum-possible welfare. The key claim is that $p_{t_1}(v^1, B^*) + p_{t'_1}(v^1, B^*) \geq 1$. Assume otherwise, and consider an instance $I'$ with the transactions $t_1$ and $t_2$ (with $s_{t_1} = s_{t_2} = 1$ and $k = 1$) but no searchers, and with $v'_{t_1} = v'_{t_2} = 1$. In $I'$, to achieve a non-trivial welfare guarantee, $M$ must include either $t_1$ or $t_2$ (and cannot include both). Assume that $M$ chooses to include $t_2$. (Otherwise, redefine instance $I_1$, swapping the roles of $t_1$ and $t_2$.) Then, if the creator of $t_1$ bids truthfully in $I'$, they receive zero utility; by DSIC (for users), no single bid would result in positive utility. But suppose instead they misreported their valuation as $1 - \epsilon/4$ while also submitting a fake bundle with a shill bid of $1 - \epsilon/4$; the resulting input to $M$ is identical to the truthful input to $M$ in $I_1$, and so $M$ will include $t_1$ at a combined price (for both the transaction and the fake bundle) of $p_{t_1}(v^1, B^*) + p_{t'_1}(v^1, B^*) < 1$, resulting in strictly positive utility for $t_1$’s creator. This contradicts the assumed sybil-proofness of $M$ and proves the key claim.

Now define the instance $I_2$ by $v_{t_1} = 1 - \epsilon/4$, $v_{t'_1} = 3$, and $v_{t_2} = 1$, and denote this valuation vector by $v^2$. For $M$ to achieve at least a $\frac{1}{2} + \epsilon$ fraction of the maximum-possible welfare in $I_2$, it must choose $B^*$. Arguing as in $I_1$, because $M$ is sybil-proof and DSIC for users, $p_{t_1}(v^2, B^*) + p_{t'_1}(v^2, B^*) \geq 1$. Because $v^1_{t_1} = v^2_{t_1}$ and $M$ is DSIC for searchers, $p_{t_1}(v^2, B^*) = p_{t'_1}(v^1, B^*)$. Individual rationality of $M$ in $I_1$ implies that $p_{t'_1}(v^1, B^*) \leq 1 - \epsilon/4$, and we can therefore conclude that $z := p_{t_1}(v^2, B^*) > 0$. Because $M$ is DSIC, $z$ is the minimum bid at which $t_1$ would be included: for all bid vectors $(z', 3, 1)$ with $z' < z$, $M$ would choose a block that excludes $t_1$.

Finally, define the instance $I_3$ by $v_{t_1} = z/2$, $v_{t'_1} = 3$, and $v_{t_2} = 1$. If users and searchers bid truthfully, then, as noted above, $M$ must choose a block that excludes $t_1$ (and therefore also excludes $t'_1$). This block has welfare at most 1 which, provided $\epsilon$ is sufficiently small, is less than $1/2 + \epsilon$ times the maximum-possible welfare. This contradiction completes the proof of Theorem 5.7 for the case of large transactions.

To extend the proof to the case of arbitrarily small transactions, choose an arbitrarily small
value for $\epsilon > 0$ and an arbitrarily large block size $k$. We consider two types of transactions: type 1 transactions $t$ with $v_t = v_{t\ast} = 1 - \epsilon/4$, and type 2 transactions $t$ with $v_t = 1$ and $S_t = \emptyset$. Assume that $s_t = s_{t\ast} = 1$ for both types, so that every transaction or bundle consumes at most a $1/k$ fraction of a block. Suppose for contradiction that $\mathcal{M}$ is a deterministic TFM that is DSIC for users and searchers, is sybil-proof, and (assuming truthful bids) always results in welfare at least a $1/2 + \epsilon$ time the maximum possible.

Let $I_1$ be an instance with $k$ type 1 transactions and $2k+1$ type 2 transactions. Let $v^1$ denote the corresponding valuation vector. Let $B_1$ denote $x(v^1, B)$. Because $\mathcal{M}$ guarantees at least a $1/2 + \epsilon$ fraction of the maximum-possible welfare, $B_1$ includes some type 1 transaction $t_1$ (along with $t^\ast_t$). We claim that $p_{t_1}(B_1, v^1) + p_{t^\ast_1}(B_1, v^1) \geq 1$. Suppose not, so that $p_{t_1}(B_1, v^1) + p_{t^\ast_1}(B_1, v^1) < 1$, and consider an instance $I_2$ that is identical to $I_1$ except that $t_1$ is converted from a type 1 to a type 2 transaction. Denoting the valuation vector in $I_2$ by $v^2$ and $x(v^2, B)$ by $B_2$, we must have $t_1 \in B_2$ and $p_{t_1}(B_2, v^2) \leq p_{t_1}(B_1, v^1) + p_{t^\ast_1}(B_1, v^1) < 1$; otherwise, in $I_2$, the creator of $t_1$ would be incentivized to sybil as a type 1 transaction (i.e., to report a false bid of $1 - \epsilon/4$ and also participate as a fake searcher with bid $1 - \epsilon/4$). Now consider a third instance $I_3$ that is identical to $I_2$ except with $t_1$ removed. Denote the valuation vector for $I_3$ by $v^3$ and $x(v^3, B)$ by $B_3$. Apart from $t_1$, $I_2$ and $I_3$ have the same set of $2k+1$ type 2 transactions. Thus, there must be some type 2 transaction $t_2$ in both $I_2$ and $I_3$ such that $t_2 \notin B_2$ and $t_2 \notin B_3$. But then, in $I_3$, the creator $i$ of $t_2$ could submit an additional copy of its transaction as $t_1$ (with bid 1), resulting in the same outcome $B_2$ as in $I_2$. Because $t_1 \in B_2$ and $t_2 \notin B_2$, agent $i$'s utility for this outcome is $v_i - p_{t_1}(B_2, v^2)$. As $v_i = 1$ and $p_{t_1}(B_2, v^2) < 1$, this utility is strictly positive and therefore more than $i$ would receive from a truthful bid (or, by DSIC, any single bid) under its true identity. This contradicts the assumed sybil-proofness of $\mathcal{M}$ and proves the claim: it must be that $p_{t_1}(B_1, v^1) + p_{t^\ast_1}(B_1, v^1) \geq 1$.

Next, define instance $I_4$ as identical to $I_1$ except with $v^*_{t_1} = 4k$. Denote the corresponding valuation vector by $v^4$ and $x(v^4, B)$ by $B_4$. Because $\mathcal{M}$ guarantees at least a $1/2 + \epsilon$ fraction of the maximum-possible welfare, $t^*_{1} \in B_4$. Arguing as for $I_1$, the fact that $\mathcal{M}$ is both DSIC and sybil-proof implies that $p_{t_1}(B_4, v^4) + p_{t^\ast_1}(B_4, v^4) \geq 1$. Because $\mathcal{M}$ is DSIC (for searchers), $v^*_{t_1} = v^1_{t_1}$, and $t^*_{1} \in B_1 \cap B_4$, $p_{t_1}(B_4, v^4) = p_{t^\ast_1}(B_1, v^1)$. Individual rationality of $\mathcal{M}$ in $I_4$ implies that $p_{t_1}(B_1, v^1) \leq 1 - \epsilon/4$, and we can therefore conclude that $z := p_{t_1}(B_4, v^4) > 0$. Because $\mathcal{M}$ is DSIC, $z$ is the minimum bid at which $t_1$ would be included: for all bid vectors $(z', v^4_{t_1})$ with $z' < z$, $\mathcal{M}$ would chose a block that excludes $t_1$.

Finally, define an instance $I_5$ that is identical to $I_4$ except that $v_{t_1} = z/2$. As noted above, if users and searchers bid truthfully, $\mathcal{M}$ must chose a block that excludes $t_1$ and hence also $t^*_{1}$; such a block has welfare at most $2k - 1$. Because any outcome that includes $t_1$ and $t^*_{1}$ achieves welfare at least $4k$, this contradicts the assumption that $\mathcal{M}$ guarantees at least a $1/2 + \epsilon$ fraction of the maximum-possible welfare. This contradiction shows that $\mathcal{M}$ cannot exist, completing the proof.

References


