# Climate Change Policy: Quantifying Uncertainties for Damages and Optimal Carbon Taxes

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## Abstract

Controversy surrounds climate change policy analyses because of uncertainties in climatic effects, impacts, mitigation costs and their distributions. Here

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we address uncertainties in impacts, and provide a method for quantitative estimation of the policy implications of such uncertainties.

To calculate an "optimal" control rate or carbon tax a climate-economy model can be used on estimates of climate damages resulting from warming scenarios and several other key assumptions. The dynamic integrated climate-economy (DICE) model, in its original specification, suggested that an efficient policy for slowing global warming would incorporate only a relatively modest amount of abatement of greenhouse gas emissions, via the mechanism of a small (about $5 per ton initially) carbon tax. Here, the DICE model is reformulated to reflect several alternate published estimates and opinions of the possible damages from climatic change. Our analyses show that incorporating most of these alternate damage estimates into DICE results in a significantly more aggressive optimal policy than that suggested by the original model using a single damage function. In addition, statistical distributions of these damage estimates are constructed and used in a probabilistic analysis of optimal carbon tax rates, resulting in mostly much larger (but occasionally smaller) carbon taxes than those of DICE using point values of damage estimates. In view of the large uncertainties in estimates of climate damages, a probabilistic formulation that links many of the structural and data uncertainties and thus acknowledges the wide range of

"optimal" policies is essential to policy analysis, since point values or "best guesses" deny policy makers the opportunity to consider low probability, but policy-relevant, outliers. Our presentation is offered as a prototypical example of a method to represent such uncertainties explicitly in an integrated assessment.

*Keywords:* Optimal carbon tax; Climate policy; Greenhouse gas abatement; Integrated assessment of climate change

# Introduction

With increasing evidence that anthropogenic emissions of greenhouse gases will lead to an increase in surface temperature, increased hydrologic extremes, and possibly several other climatic effects [Houghton et al., 1996], research has been increasingly focused on the potential impacts of climatic change.

Some express deep concern over the possibility of catastrophic damage from changes in ocean circulation, a melting of the West Antarctic Ice Sheet, and other improbable but plausible changes in climate-related systems [Broecker, 1997]; [Schneider et al., 1998]. Concerns for significant climate damage motivates advocacy for reducing the amount of global greenhouse gas emissions (especially carbon dioxide, $CO_2$, the most important greenhouse gas), to below so-called "business as usual" (BAU) baseline projections [Azar and Rodhe, 1997]. The primary proposed mechanism is by economic incentives — typically a carbon tax — to promote less carbon-intensive fuels and to develop alternatives (e.g., [Schneider and Goulder, 1997] discuss costs of a carbon tax versus direct research and development subsidies). Others seem confident that humans will be largely capable of adapting to most projected changes and argue that short-term growth should not be restricted much to reduce climatic change [Schelling, 1983].

There are many uncertainties in estimated climatic change effects, impacts, and costs of carbon abatement. Moreover, there is little agreement on how to place a dollar value on the non-market impacts of climatic change, such as the loss of human life, biodiversity, or ecosystem services. There is also debate regarding what kinds and what values of discount rates should be applied in cost-benefit studies [Chapman et al., 1995]; [Hasselmann et al., 1997]; [Nordhaus, 1997]. Nevertheless, several studies suggest that climatic change will have only minor economic impacts, and that an optimal policy would therefore incorporate only modest controls on greenhouse gas emissions [Kolstad, 1993]; [Nordhaus, 1991]; [Peck and Teisberg, 1992]. However, many of these "modest controls" conclusions are based on point estimate values — that is, results which are derived from a series of "best guesses". This point estimate method fails to account for the wide range of plausible values for many parameters. Since policy making in the business, health and security sectors often is based on hedging against low probability but high consequence outcomes, any climate policy analytic tools that represent best guess point values or limited (i.e., "truncated") ranges of outcomes restrict the ability of policy makers to make strategic hedges against such risky outlier events. In this paper, we demonstrate a method to include the wide range of outcomes and show their implications for climate policy.

Clearly, given the many uncertainties and unexplored assumptions in conventional economic analysis [Azar, 1996]; [Brown, 1997]; [Grubb, 1997]; [Jenkins, 1996]; [Repetto and Austin, 1997]; [Schneider, 1997], it is necessary to formally develop and apply a variety of methods to explore a range of possible conclusions. Thus, we will demonstrate in explicit detail here quantitative methods that can be used to explicitly incorporate a wide range of uncertainty estimates (including outliers) of the impacts of climatic change. In addition, little attention has been given to the non-market impacts of climatic change, and the implications of modeling market and non-market impacts separately [Daily, 1997]; [Tol, 1995]. However, some studies have tried to incorporate aspects of such uncertainties [Manne and Richels, 1994]; [Morgan and Dowlatabadi, 1996]; [Nordhaus, 1994b]; [Peck and Teisberg, 1995], finding that their inclusion typically increases the magnitude of "optimal" abatement efforts. This study goes beyond these previous attempts by including the full range of climate damage estimates (i.e., not truncating the distribution to exclude outlier estimates) and/or by performing a Monte Carlo simulation based on published climate damage distributions, which yields a statistical distribution of optimal policy responses.

We focus on key assumptions made in earlier analyses — specifically, in the pioneering model of Nordhaus (DICE) — and determine the sensitivity of the model's conclusions to plausible alternative assumptions. In particular, we consider the climatic change impact estimates published by several researchers and experts, and construct several reformulated DICE models to reflect a variety of these estimates and opinions. We then compare the results and policy implications of the reformulated models to those of the previous studies. We conclude that the policy community must be sure that a wide range of estimated outcomes are explicitly represented in any policy analysis so that strategic hedging may be one of the policy options considered.

## Integrated Assessment

There are several Integrated Assessment models (IAMs) that have been used for the analysis of emission control policies. These models vary in complexity, structure, and the numerical values of key parameters. Indeed, no IAM can credibly deal with all important factors nor cover the wide range of value-laden alternatives that need to be considered in real-world policy-making (for example, see the review by [Schneider, 1997] or the special issue edited by [Filar et al., 1998]). Nevertheless, IAMs can provide insights via sensitivity analyses of key uncertain parameters, structural elements, and value choices. The DICE model is a well known, well documented and relatively simple IAM. The transparency of the model allows for several reformulations and extensions, which will be important for our purposes of displaying quantitative methods of presenting uncertainties and demonstrating that policy makers need to be aware of the full range of potential outcomes. Although the simplicity of this approach precludes taking the quantitative results literally, the qualitative insights from our presentation will hopefully prove useful to the climate policy-making community.

## The DICE Model

All of the quantitative analyses here use the Dynamic Integrated model of Climate and the Economy (the DICE model), as described by Nordhaus [Nordhaus, 1992]. A short overview of the model is given in Appendix A. Put briefly, DICE optimizes the trade-off between the costs of climatic change and the costs of restricting greenhouse gas emissions. Here we only reconsider the former cost, and examine DICE's sensitivity to the formulation of the damage term (equation A8 in Appendix A) with several alternate damage functions. One attempt to quantify the importance of the damages of climatic change appears in [Nordhaus and Popp, 1997]. That study demonstrates that improved estimates of climate damages are more valuable than improved estimates of any other parameter in the DICE model. This paper extends their conclusion by adding a sensitivity analysis of DICE policy conclusions to alternate published damage estimates (including outliers which might imply strategic hedging policies), thereby providing a wide range of plausible optimal policies in the presence of uncertainty in the magnitude of climatic damage.

The DICE model was originally designed to compare the economic effects of several different policies regarding the control of anthropogenic carbon dioxide emissions. One such policy is a "Business-As-Usual" (BAU) or baseline scenario, where no efforts are made to control greenhouse gas emissions. In terms of equation A5 in Appendix A, this scenario fixes the endogenous control rate $\mu(t) = 0$ for all $t$. We will sometimes refer to this as the "no-controls" constraint. In this scenario, there is no emissions abatement (and thus no abatement costs), but we anticipate the temperature increase, and hence the damage from climatic change, to be higher.

In the "optimal" policy scenario, the "no-controls" constraint is relaxed, and $\mu(t)$ is determined endogenously. In this scenario, DICE is free to trade off the costs of climatic change with those of emissions abatement. If the costs of global warming are relatively small, the incentive to mitigate carbon emissions will also be small, so we would expect $\mu(t)$ to be close to zero (i.e., the BAU scenario would be close to the optimal one). If the impacts of climatic change are great, however, we would expect $\mu(t)$ to be much closer to 1, implying that "Business-As-Usual" would be a relatively poor policy, from the point of view of optimizing economic efficiency.

Discounted consumption is used by Nordhaus as the primary criterion for comparing different model results. "Discounted consumption" here refers to all consumption occurring after 1989, discounted to 1990 by the rate of interest on goods and services calculated in the standard optimal DICE run. Since utility is an increasing function of consumption, in this formulation larger quantities of discounted consumption are taken as more desirable — a premise challenged by some (e.g., [Brown, 1997]; [Jenkins, 1996]) but accepted here for the purposes of our analysis and to demonstrate policy implications of including a wide range of outcomes.

Discounted consumption can also serve as an indicator of the severity of climatic change (since unmitigated warming will decrease production, and hence consumption). Because of the large scale (hundreds of trillions of dollars) for discounted consumption from 1990 to 2100, sometimes different formulations of climatic damage will have little impact (percentagewise) on total discounted consumption, even though the absolute evolving differences over time can be quite large and thus have important short and long-term policy ramifications.

Optimal emission control rates, the values of $\mu(t)$ for the optimizing run of a model, are a second important indicator of the consequences of climatic damage. An "optimal policy" is one which uses these values of $\mu(t)$. This optimal policy can be achieved via an "optimal carbon tax" value. In other words, we are interested in the level of carbon tax that would induce the optimal values of $\mu(t)$.

The optimal carbon tax can be calculated as a ratio of "shadow prices" (or "dual variables") of the model's non-linear program. The shadow price of consumption ($C_m$) is equivalent to the increase in the model's objective function (equation A1 in Appendix A) from one additional unit of consumption (relative to the optimal level), and the shadow price of carbon emissions ($E_m$) represents the increase in utility from one additional unit of carbon emissions. Thus, by taking the ratio of these shadow prices ($E_m/C_m$), we derive the implicit price of carbon per unit of consumption, which provides a calculation of the carbon tax.

# Alternate Damage Functions

There are currently many different estimates and countless opinions regarding the economic impacts of global warming. The DICE model includes a climate damage function based on Nordhaus's personal estimate. We first compare this function with those of several other damage estimation studies, and later add in the opinions of eighteen experts surveyed by Nordhaus in a subsequent study [Nordhaus, 1994a].

## Nordhaus's damage function

In deriving a damage function for his DICE model, Nordhaus first estimated the effect of a 3°C warming on U.S. income. Based on the results in [Nordhaus, 1991], Nordhaus used a 0.25% loss of GDP as the starting estimate for this value. Due to the difficulty in quantifying all of the probable damage from climatic change, especially non-market damage, this may be an overly conservative guess. To account for this, Nordhaus raised the estimate to 1% of U.S. income. This value may still be too conservative for a global model (such as DICE), since damage estimates for the United States are likely to be considerably less than those for countries which have a greater dependence on agriculture and a more limited ability to adapt to climatic change (as is the case with many less developed countries [Bruce et al., 1996]). A second adjustment was then made to extrapolate a global estimate from the domestic estimate, and a total (negative) impact of 1.33% to global output was used by Nordhaus for a 3°C warming in the DICE model.

We are also interested in the relationship between damage and warming as warming increases beyond the 3°C value. Recognizing that disproportionally larger damages have been hypothesized for larger climate changes than for smaller ones, Nordhaus assumed a quadratic function. This yields the following damage function for the original DICE model:

$$d_N(t) = 0.0133[\Delta T(t)/3]^2 \tag{1}$$

where $d_N(t)$ is the fractional loss of global output, and $\Delta T(t)$ is the rise in average global temperature.

## Alternate Damage Functions

Other experts have made independent estimates of the damage of global warming. This section uses the results of several of these studies to derive alternative damage functions for use in the DICE model.

Table 1 presents an overview of recent damage estimates for a doubling of $CO_2$ levels by Cline, Fankhauser, Titus, and Tol [Cline, 1992]; [Fankhauser, 1995]; [Titus, 1992]; [Tol, 1995]. Detailed breakdowns of these estimates have been published by the IPCC [Bruce et al., 1996]. However, these values only consider damage to the United States, and only describe a damage function for a single temperature increase value. To derive continuous functions consistent with DICE, we borrow the assumptions from Nordhaus's approach — that total damage to the global output will be a factor of one-third greater than the damage to United States output, and that damage is a quadratic function of global warming, with zero damage for an unchanging climate. Under these assumptions, the damage functions for these four estimates are as follows.

$$d_C(t) = 0.0146[\Delta T(t)/2.5]^2 \tag{2}$$

$$d_F(t) = 0.0173[\Delta T(t)/2.5]^2 \tag{3}$$

$$d_{Ti}(t) = 0.03325[\Delta T(t)/4.0]^2 \tag{4}$$

$$d_{To}(t) = 0.0200[\Delta T(t)/2.5]^2 \tag{5}$$

The relative character of all five damage functions is shown in Figure 1a. The functions spread out considerably with more than 3°C of warming. The function used in the original DICE model is the most conservative of the five.

[Figure 1 here]

## IPCC Damage Functions in DICE

Figure 1b presents the loss of discounted consumption due to climate damage in the "Business As Usual" (BAU) policy scenario, where no action is taken to mitigate the buildup of atmospheric greenhouse gases. These curves also represent the gross benefit of complete climatic change abatement associated with each damage function in the DICE model (since complete abatement would reduce the damage from climatic change to zero). Since a relatively large amount of climate-induced damage results in less income, and hence less consumption, the more severe damage functions result in a greater losses of discounted consumption, and hence a larger economic incentive for climatic change abatement.

We next consider the "optimal" policy scenario (i.e., we remove the "no-controls" constraint), in order to compare the levels of optimal emission control rates (the level of mitigation) and optimal carbon taxes (the mechanism used to induce the mitigation) for each of the damage estimates. Figure 1c shows the values of the optimal carbon taxes associated with each damage function (equations 1 through 5).

With the original damage function (equation 1), the DICE model calculates modest carbon taxes — less than 10 1990 U.S. dollars per ton of carbon over the next two decades, with a tax of just over 20 dollars by the end of the 21st century. By contrast, these numbers double when the DICE model is run with the damage estimates of Fankhauser or Tol (equations 3 and 5). Similarly, the optimal emission control rates for model runs with Fankhauser or Tol damage estimates are over 50% higher than those in the canonical DICE run.

## Damage Functions from an Expert Survey

A second source for estimates of damage from climatic change is an expert survey conducted by Nordhaus [Nordhaus, 1994a]. 19 experts from the natural sciences, the social sciences, technology, and economics were questioned about the economic impacts, distributional effects, and non-market effects of global warming. For all questions, three scenarios were considered: a 3°C warming by 2090 (scenario A); a 6°C warming by 2175 (scenario B); and a 6°C warming by 2090 (scenario C). Here we concentrate on the experts' opinions regarding economic impacts in scenarios A and C. (This data is shown in Table 2 [Nordhaus, 1993]. Respondent 19 is not included because he did not complete this portion of the survey.)

## Disciplinary Background Affects Damage Functions

The survey respondents were categorized by Nordhaus as natural scientists, environmental economists, and "other social scientists" (a group composed primarily of "mainstream" economists). As the final rows of Table 2 show, there is considerable variation in opinion between researchers of these different fields. The natural scientists' average damage estimate is far more pessimistic for the world economy than that of the social scientists, and the environmental economists average an estimate between the other two. Additionally, Figure 2 plots climate damage estimates versus market damage estimates for all respondents except the "outliers" (i.e., excluding two respondents who estimated less than a 0.3% loss of GWP from 3°C of warming, and two respondents who estimated more than a 15% loss of GWP from 3°C of warming). Figure 2 shows that respondents who estimated a large amount of climate damage were more likely to place a larger proportion of those damages outside of the standard national accounts (*i.e.,* large damage estimates implied large non-market damages).

[Figure 2 here]

The differences across disciplines in the survey results have been previously noted [Nordhaus, 1994a]; [Peck and Teisberg, 1995]. Indeed, Peck and Teisberg have used the survey respondents' estimates of a "high-consequence outcome" (one defined as a sustained loss of global output of 25% or more) to incorporate risk into the CETA model [Peck and Teisberg, 1995].

Some researches have argued that any decision analytic survey in which groups of respondents appear to hold to different paradigms should avoid aggregating experts; the estimates of each paradigmatically different group should be presented separately [Keith, 1996]; [Morgan and Henrion, 1990]; [Pate-Cornell, 1996]. Unfortunately, when the survey participants are grouped together by discipline, the smaller pools (e.g., natural scientists) consist of only a few individuals. Because of the small sample size, much of our analysis aggregates the estimates of all of the experts, despite the strong caveat (but in Figure 3 we do show a "traceable account" of the subgroups that we subsequently aggregate into one summary distribution in Figure 4). On the other hand, drawbacks of considering each paradigmatically distinct group separately are pointed out in [Titus, 1997].

The data from Nordhaus's survey can be used to derive several new damage functions. In particular, we can use the estimates from scenarios A and C as two data points (for $\Delta T(t) = 3$ and $\Delta T(t) = 6$, respectively). With our previous assumption of zero damage for a zero-warming scenario, we have a third data point at $\Delta T(t) = 0$. We can then derive a unique continuous damage function for each set of three data points. One should note that the assumption of a quadratic relationship between damage and warming that we borrowed from Nordhaus and used above is now relaxed. That is, unlike our previous "assumed quadratic" functions, the exponents of these "curve fit" damage functions follow directly from the data. However, this analysis still only considers functions of a single term of the form $ax^b$. A dual-term approach to quantifying damage has been discussed in [Roughgarden, 1997].

There, the DICE model was reformulated to include one market damage term and one non-market damage term. Non-market damage affected global utility directly, rather than indirectly through income (as is the case in the original specification of DICE). Preliminary analyses suggest that the DICE model is much more sensitive to the magnitude of the damage function than to a partitioning of the damage function into market and non-market components. A similar, less extensive analysis appears in [Tol, 1994].

We begin by deriving damage functions for each of the disciplines represented in the survey. The 50th percentile estimates from Table 2 yield the following damage functions for the natural scientists, the environmental economists, the other social scientists, and the entire group of respondents:

$$d_{NS}(t) = 0.0231\Delta T(t)^{1.57} \tag{6}$$

$$d_{EE}(t) = 0.0218\Delta T(t)^{1.01} \tag{7}$$

$$d_{SS}(t) = 0.0022\Delta T(t)^{1.87} \tag{8}$$

$$d_{All}(t) = 0.0067\Delta T(t)^{1.53} \tag{9}$$

These functions are shown graphically in Figure 3a, together with the original DICE damage function for comparison. The DICE damage function is similar to that of the social scientists, and is substantially more optimistic than the other three functions.

It is interesting to note that differences of opinion show up primarily in the coefficients of these functions, rather than in the exponents. For example, equation 7 (for the environmental economists) is nearly linear and equation 8 (for other social scientists) is nearly quadratic. However, this fact is overshadowed by the order of magnitude difference between the two equations' coefficients — in the warming range that we are interested in, the former equation has a much greater value than the latter. Overall, the experts seem to largely agree that there is a nonlinear relationship between damage and warming, but that there is less than quadratic dependence on

$\Delta T(t)$. This contrasts to the less than linear damage functions in a survey of non-expert home owners in California [Berk and Schulman, 1991].

The second group of functions compares the low, middle, and high estimates of the respondents. Here, we disregard the backgrounds of the participants and concentrate solely on the spread of all respondents' aggregated estimates. The following equations are derived from the aggregated averages for the 10th, 50th, and 90th percentile damage estimates:

$$d_{10}(t) = 0.0006\Delta T(t)^{2.24} \tag{10}$$

$$d_{50}(t) = 0.0067\Delta T(t)^{1.53} \tag{11}$$

$$d_{90}(t) = 0.0164\Delta T(t)^{1.44} \tag{12}$$

Note that, by definition, equations 9 and 11 are identical.

[Figure 3 here]

## Survey Damage Functions in DICE

Next, we discuss the effects of replacing the original DICE damage function (equation 1) with equation 6, the natural scientists' 50th percentile response, and with equation 12, the entire survey group's 90th percentile estimate of damages. This "optimistic vs. pessimistic" contrast (see Figure 3b) is useful for examining the importance of the large differences in opinion regarding the potential impacts of climatic change on policy (i.e., optimal carbon taxes).

The first analysis of DICE with each of the above three damage functions compares the net present value of consumption after 1990 in the BAU policy scenario, where no efforts are made to slow climatic change. In this case, we would expect the model runs with pessimistic damage functions to exhibit decreased consumption, due to increased damage from unmitigated global warming. Reformulating DICE with the damage function given by equation 12 results in a 2.8% loss in discounted consumption, and using the damage function given by equation 6 causes a 4.4% loss in discounted consumption (relative to a world without climate-induced damage). Thus, the benefit of emissions control in the reformulated DICE models is several times higher than that in the original DICE model (where a BAU policy resulted in a less than 0.5% loss in discounted consumption). We examine the effects of these incentives to abate carbon by rerunning the DICE model and including the possibility of policies that restrict carbon emissions (i.e., we consider the "optimal" policy scenario).

Using the damage function from the 90th percentile estimates of all of the experts, DICE calculates about three times as much carbon taxes as in the more optimistic scenario. With the damage function based on natural scientists' 50th percentile estimates, optimal carbon taxes (Figure 3b) are about six times as large as those in the optimistic scenario. The values of optimal control rates for these two models (Figure 3c) range from two to three times higher than those in the original model. The average of the median estimates (i.e., equation 11) yields results closer to those of the DICE model, but still gives higher values for optimal control rates and carbon taxes.

Finally, several researchers argue that a low (or even zero) rate of social time preference is appropriate for the DICE model, on the basis that it is philosophically indefensible to value the welfare of future generations less than the welfare of the present generation (even if this yields a discount rate inconsistent with observed economic behavior, such as the global savings rate) [Azar and Sterner, 1996]; [Cline, 1992]. Thus, as a pure sensitivity analysis comparison, in Figure 3c we include a curve in which the 3% social rate of time preference of DICE ($\rho$ in equation A1 of Appendix A) is replaced by a smaller rate (1.5%). We also use equation 11 (i.e., the aggregate median) as a damage function. The resulting increase in optimal emission control rates is larger than that caused by using the experts' 90th percentile damage estimates (as reflected in equation 12) in place of the experts' median estimates (equation 11), showing a high sensitivity to a small change in $\rho$.

A more thorough sensitivity analysis of the DICE model to the social rate of time preference was performed in [Chapman et al., 1995]. This study found that a zero rate of social time preference leads to an optimal control rate almost three times that of the original DICE model. Additionally, it shows that replacing the decreasing function $\sigma(t)$ in equation A5 of Appendix A (the $CO_2$-equivalent emissions per unit of output without controls) with a constant function causes a similar increase in optimal control rates. Finally, the study by [Kaufmann, 1997] demonstrates that alternate assumptions about the transfer of carbon from the atmosphere to the ocean yield increased climate damages, and hence a more stringent optimal policy. However, our purpose here is not to address the structural assumptions or parameters in DICE that are both debatable and have large impacts on policy options, but to concentrate on damage function sensitivity and the implications of ignoring outliers.

## Subjective Probability Distributions of Survey Respondents

To this point we have derived damage functions for particular estimates of the respondents to Nordhaus's survey. However, this approach does not capture all of the information in the results of the survey. Each respondent gives a subjective probability distribution for damage in each warming scenario, rather than simply point estimates. Thus, we can combine these distributions and construct an aggregated "expert probability distribution" for damage from climatic change. We do not suggest that the resulting functions should be viewed as particularly "credible", as expert opinion on climate damage will likely change markedly as new research reshapes subjective opinions [Schneider, 1997]. Further, this "aggregate expert opinion" should not be considered a "consensus" among experts, since several survey respondents would undoubtedly strongly disagree with the properties of the aggregate damage distributions. We do, however, believe that this analysis technique in which uncertainties are explicitly displayed provides much better insights to policy-makers viewing integrated assessments than simple point values. Moreover, it is useful to look at the implications of the spread of opinions in considering decision-making under uncertainty of the severity of climatic damage, in particular the opportunity to consider strategic hedging policies to deal with extreme event possibilities.

In Nordhaus's survey, each respondent gives low (10th percentile), median or "best guess" (50th percentile), and high (90th percentile) estimates for damage in each scenario. As before, we restrict our attention to scenario A (a 3°C warming by 2090) and scenario C (a 6°C warming by 2090). One obvious distribution to consider is the symmetric normal distribution. However, the skewness of the estimates must be considered. An easy way to check for skewness is to compare each best guess estimate to the average of the respective low and high estimates. If a respondent's estimates are symmetric, these two values should be equal — that is, the chance of overestimating climatic damage by a given amount should be the same as that of underestimating damage by the same amount.

Referring back to Table 2, we see that, of the 36 sets of 10th, 50th, and 90th percentile estimates, only 5 sets of estimates suggest a symmetric distribution (respondent 16 in scenario C, and respondents 15 and 18 in both scenarios). Four sets of estimates exhibit left-skewness (a best guess estimate which is greater than the average of the low and high estimates), and the remaining 27 sets of estimates exhibit right-skewness (a best guess estimate which is smaller than the average of the other two). Thus, the bulk of the (both optimistic and pessimistic) experts thought that their best guess for climatic damage had a greater chance of being a large underestimate than a large overestimate; in other terms, a higher probability of a "nasty surprise" than a "pleasant surprise" [Schneider et al., 1998].

Given the skewness of the data, we fit a Weibull distribution to the damage estimates of each survey respondent.

## Aggregate Damage Distributions for 3°C and 6°C Warming Scenarios

To construct an aggregate damage distribution, a range of relevant damage levels is first identified. "Relevant" is defined here as within 1.5 standard deviations of *some* expert's best estimate. Using this definition, we consider damage estimates from -2.7% to 33.9% of GWP in scenario A, and from -1.4% to 100% of GWP in scenario C. It should be noted that distributions for both warming scenarios will consider the possibility of a net benefit from climatic change (from increased agricultural yields, new discoveries of or access to minerals in the polar regions, reduced cold season health impacts, etc.).

To finish the construction of an aggregate distribution, we must simply "add up" the subjective probability distributions and normalize their sum. This requires a "discretization" of the probability distributions, as they are not easily summed in closed form. This is done by partitioning the range of relevant damage levels into 100 subranges, and splitting up each probability distribution across the subranges. Then, we simply normalize by the number of respondents (18) to ensure that the total value of the range will be equal to 1. The result can then be considered a discrete approximation to the aggregate damage distribution, with each subrange approximating the value of the probability density function (PDF) of the aggregate distribution at a single point.

More formally, we can describe this approximate aggregate distribution as follows. Denote the cumulative distribution function (CDF) of the $i$th subjective probability distribution as $F_i$, and the subrange with left endpoint $x_1$ and right endpoint $x_2$ as $S_{1,2}$. Then the value of $S_{1,2}$ is given by:

$$S_{1,2} = \frac{\sum_{i=1}^{18}(F_i(x_2) - F_i(x_1))}{18} \tag{13}$$

$S_{1,2}$ can be considered an approximation of $\tilde{f}(\frac{x_1+x_2}{2})$, where $\tilde{f}$ denotes the actual (continuous) probability density function of the aggregate damage distribution.

The CDFs for damage in scenarios A and C are given in Figure 4a (labelled $F(x)$) and the PDFs are given in Figure 4b (labelled $f(x)$).

A striking feature of both distributions is their right-skewness (i.e., "surprise potential"). For the 3°C warming scenario, the mode of the distribution (the peak of the PDF) is very close to zero, indicating that for this statistic aggregated expert opinion suggests that the benefits of climatic change are likely to offset most of the costs. Looking at the CDF, however, we see that there is a significant ($> 10\%$) chance of a loss of more than 10% of the gross world product in this scenario. For scenario C, the shape of the distribution is similar. According to the aggregated expert opinion, there is a 50% chance of experiencing less than 6% GWP loss from 6°C of warming, but a 4% chance that the climatic change in this scenario will cut global output in half — an unfathomable economic catastrophe!

Nordhaus also used right-skewed damage distributions in his formal sensitivity analysis of the DICE model [Nordhaus, 1994b] based on the expert opinion expressed in his survey [Nordhaus, 1994a]. Our analysis differs from Nordhaus's in that we use an input distribution based solely on expert opinion, rather than one centered around Nordhaus's personal damage estimate. In addition, we consider the damage estimates of all 18 survey respondents, whereas Nordhaus ignored the "outliers", considering only the "trimmed mean" of the survey results (a statistic that ignored the three highest and three lowest estimates, markedly changing the output distribution). Moreover, since all of the participating natural scientists were among the pessimistic outliers, our distributions reflect the beliefs of a group not represented by the damage distributions of [Nordhaus, 1994b]. Including the opinions of natural scientists in the construction of the damage distributions yields an increased asymmetry in the probability density functions and higher expected damages given a particular temperature increase. This in turn increases the expected value of both optimal control rates and optimal carbon taxes. Whose opinions will turn out to be more credible is empirically testable, of course, by "performing the experiment" of substantial climate change over the next century. Whether or not to take that risk is a value judgement we won't confront in this article, but one that policy makers contemplating ratification of the Kyoto protocol will have to confront (for the personal views on this subject by one of the authors, see [Schneider, 1998]).

[Figure 4 here]

## Damage Functions from Distributions

The general approach for deriving a damage function from random damage estimates is similar to the one outlined earlier: given damage estimates for $\Delta T(t) = 3$ and $\Delta T(t) = 6$, assume no damage for $\Delta T(t) = 0$, and derive a function through the three data points of the form $ax^b$. In most cases, the procedure is identical to the one used above to derive damage functions from the point estimates of the survey respondents; the details are deferred to Appendix B.

Five damage functions derived from random samples of the damage distributions are presented in Figure 4c. For contrast, damage functions from the

1st, 10th, 50th, 90th, and 99th percentile damage estimates are shown. The 1st and 10th percentile damage functions were derived from equations B4 through B7 in Appendix B (since they must pass through negative damage estimates), while the other three functions are derived from equations B1 through B3 in Appendix B.

# Results from a Monte Carlo Simulation

Using the methods of the previous section, we can now perform a probabilistic analysis with the expert opinions from Nordhaus's survey. This section discusses the results of a Monte Carlo simulation, a simulation which generates data from a series of "runs". In this analysis, each of one thousand runs selects random input parameters, drawn from the previously derived damage distributions, reformulates the DICE model with a damage function derived by the method outlined in Appendix B, and runs the new model to generate data for the optimal and BAU scenarios. This exercise is useful for evaluating the effects of the uncertainty of the economic costs (market and non-market, as both were implicit in Nordhaus's survey) of climatic change on the output of the DICE model. In particular, the data from the simulation runs yield an output distribution, which will allow a comparison between the standard DICE model and the opinions expressed in Nordhaus's survey. This comparison is more comprehensive than that for Figure 3, where the analyses relied solely on specific point estimates of expert opinion, rather than on entire subjective probability distributions.

## Optimal Carbon Taxes

We have already seen that the damage distributions derived from the aforementioned expert survey have large variances in the magnitude of damage from unmitigated climatic change. We now consider the distribution of optimal policy, in the form of carbon taxes, associated with these damage distributions.

Using the results of the Monte Carlo simulation, distributions for optimal carbon taxes in the years 1995, 2055, and 2105 were derived. The CDFs for these distributions are shown in Figures 5a, 5c, and 5e, and the PDFs are given in Figures 5b, 5d, and 5f. Points showing the optimal carbon taxes calculated by the original DICE model are shown for comparison.

[Figure 5 here]

All three distributions show a heavy concentration of results near 0. For the year 1995, nearly a quarter of the simulation runs give an optimal tax level less than that of the original DICE run (5.24 1990 US dollars per ton of carbon). For 2055 and 2105, this fraction of relatively optimistic runs is slightly higher (where the original DICE model gave optimal carbon taxes of 15.04 and 21.73 1990 US dollars, respectively).

However, all three optimal carbon tax distributions suggest a non-negligible probability that a large carbon tax is needed for optimal response to potential climatic change. One quarter of the simulation runs "recommend" a 1995 carbon tax of at least $50 per ton of carbon, which is a tenfold increase from the optimal tax in the canonical run. About 15% of the runs give similarly enhanced tax levels for 2055 and 2105. In the most pessimistic damage runs, optimal carbon taxes start at nearly $200 per ton in 1995, and climb to nearly $500 per ton by the end of the 21st century. We reiterate that all of these carbon tax rates are "optimal", and differ only by the damage function assumed.

## Comparison of Results with DICE

Several comparisons between our optimal carbon tax distributions and the output of the original DICE model can be made, using the data summarized in Table 3 and Figure 5.

Comparing the mode (the most frequent value) of the output distribution with the results of the original DICE model, it seems that DICE is a good representative of the expert opinion expressed in Nordhaus's survey. The modes of the optimal carbon tax distributions are near zero, close to DICE's recommendation for a relatively light carbon tax. However, the other properties of the output distributions justify very different policies. The median and mean of the optimal carbon tax distributions range from three to eight times as high as those featured in the original DICE run.

The differences between the modes of the output distributions and their medians and means can be attributed to their lack of symmetry. As a result of the preponderance of right-skewness of the opinions given in Nordhaus's survey, discussed earlier (e.g., Figure 4), the output distributions include a non-negligible probability of extremely severe damage from climatic change. These long, heavy tails (which we label "Surprise" in Table 3) pull the medians and means of the distributions away from the modes.

The differences between the output distribution and the results of the original DICE model are particularly obvious when we consider the tails of the output distributions. We take the 95th percentile results from the output distributions as representatives of these tails. Referring back to Table 3, we see that the "surprise" estimates for optimal carbon taxes are over twenty times the level of those in the canonical run.

These differences are caused by two different effects. First, the means of these distributions (4.04% and 11.22% of GWP damage for scenarios A and C, respectively) are much higher than the damage estimates used in DICE (1.33% and 5.32%). Thus, most of our Monte Carlo runs will use more pessimistic damage functions than that of the original DICE model. Second, the non-linearities of the model will, on average, push optimal carbon taxes even higher. Intuitively, damage functions derived from these damage distributions will never cause far more optimistic results than those with the original DICE damage function, but they will occasionally result in far more pessimistic outcomes. These occasional "catastrophic" damage functions will lead to a relatively pessimistic *expected value* of output. In other words, the significant chance of a "surprise" [Schneider et al., 1998] causes a much higher level of "optimal" abatement, relative to the original DICE formulation.

In addition, we analyzed the effects of the relative severity of the average survey damage estimate versus those of the non-linearities of the DICE model in a probabilistic analysis. Approximately one third of the difference between between the optimal carbon taxes of DICE and the means of our optimal carbon tax distributions are accounted for by the relatively high survey damage estimates, and the remaining two-thirds of the difference can be attributed to the non-linearities in the model.

# Conclusions

By including a wide range of published climate damage estimates and applying them to a simple, but well-known, climate-economy integrated assessment model, we make explicit to policy makers the wide range of "optimal" climate abatement policy options that this analytic approach provides. Our analysis shows that the original DICE model is a fairly good representative of the most frequent estimates from the Monte Carlo analysis based on Nordhaus's survey of experts, but DICE with Nordhaus's original damage function is far more optimistic (i.e., suggests a lower carbon rate) than the bulk of the distribution of expert opinion. In a sense, the original DICE carbon tax may be regarded as a point estimate between the mode and median of the distribution of expert opinion. However, this point estimate ignores the chance that, as estimated by 18 experts, climatic change could cause a disastrous amount of damage, a chance that most of the survey respondents clearly consider non-negligible. In other words, output from a single model run does not display all the information available nor does it offer sufficient information to provide the insights needed for well-informed policy decisions. One cannot simply look at a recommendation for a "five dollars per ton carbon tax" and claim that higher carbon taxes are "necessarily less economically efficient". As we have shown, such a relatively low carbon tax results from using a relatively optimistic damage estimate, and from ignoring the uncertainty of the magnitude of impacts from climatic change. Instead, a wide range of possible scenarios, including low-probability, beneficial and high-risk scenarios, must be explicitly considered. In particular, strategic hedging policies to deal with the 95th percentile, high damage outcome may well be chosen by policy makers, just as individuals or firms purchase insurance against low probability catastrophic outcomes. Regardless of the risk proneness or risk averseness of the individual decision maker, the characterization and range of uncertainties of the information provided by decision analysis tools must be made explicit and transparent to policy-makers [Moss and Schneider, 1997]. This range of uncertainty should also include estimates for the subjective probability of varying climatic effects (e.g., [Morgan and Keith, 1995]), damage estimates (e.g., this article), discount rates (e.g., [Cline, 1992]; [Chapman et al., 1995]), carbon cycle effects on $CO_2$ uptake (e.g., [Kaufmann, 1997]), and the sensitivity of the economy to structural changes such as induced technological change (e.g., [Grubb, 1997]; [Repetto and Austin, 1997]; [Goulder and Schneider, 1999]). The end result of any set of integrated assessment modeling exercises will be, as always, the subjective choice of a decision-maker [Schneider, 1997], but a more comprehensive analysis with uncertainties in all major components explicitly categorized and displayed will hopefully lead to a better-informed choice, including the options for strategic hedging against low probability, high consequence events.

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# A The DICE Model

The DICE model is an intertemporal, optimal-growth model of the global economy, first published in [Nordhaus, 1992]. More specifically, the model maximizes utility by choosing values for three decision variables (consumption, investment, and emissions control) subject to several economic and geophysical constraints. The exact form of the objective functions is given by:

$$\max U = \sum_t \frac{P(t)\ln(C(t)/P(t))}{(1+\rho)^t} \tag{A1}$$

where $U$ is discounted utility, $P(t)$ is the population size at time $t$, $C(t)$ is global consumption at time $t$, and $\rho$ is the social rate of time preference. In words, utility is a function of per capita consumption, discounted at rate $\rho$. The values of $P(t)$ are taken as exogenous, with population levels stabilizing around 10.6 billion people in the 24th century. $\rho$ is also taken exogenously as 3%.

Global output, $Y(t)$, is given by a Cobb-Douglas production function:

$$Y(t) = \Omega(t)A(t)L(t)^{1-\gamma}K(t)^{\gamma} \tag{A2}$$

where $A(t)$ represents technology, $L(t)$ is labor, $K(t)$ is capital, and $\gamma$ is capital elasticity. $\Omega(t)$ relates production to the costs of emission control and the costs of climatic change, and will be discussed below. $\gamma$ is assumed to be .25. $L(t)$ is assumed to be equal to $P(t)$, the population size. $A(t)$ is taken to be an exogenously increasing function, but with decreasing growth. In other words, as with population size, productivity is taken to be increasing but leveling off.

Global output is then endogenously divided among consumption and investment:

$$Y(t) = C(t) + I(t) \tag{A3}$$

and the level of investment affects the future capital stock:

$$K(t) = (1 - \delta_K)K(t-1) + I(t-1) \tag{A4}$$

where $\delta_K$ is the rate of depreciation of the capital stock, taken exogenously to be .10. Since the model uses time increments of 10 years, time $t$ is 10 years after time $t-1$.

The economic side of DICE interacts with the climatic side through greenhouse gas emissions. Specifically, emissions are taken to be a function of output:

$$E(t) = [1 - \mu(t)]\sigma(t)Y(t) \tag{A5}$$

where $\mu(t)$ is the endogenous control rate and $\sigma(t)$ is the $CO_2$-equivalent emissions per unit of output without controls. $\sigma(t)$ is taken exogenously as a decreasing function, due to historical trends of increasing energy efficiency and substitution for carbon-intensive fuels.

The magnitude of climatic change depends on the *stock* of greenhouse gases in the atmosphere ($M(t)$), not the flow of gases to the atmosphere (i.e., $E(t)$):

$$M(t) - M_{pre} = \eta E(t-1) + (1 - \delta_M)[M(t) - M_{pre}] \tag{A6}$$

where $M_{pre}$ is the preindustrial level of the stock, $\eta$ is the marginal atmospheric retention ratio, and $\delta_M$ is the rate of transfer of carbon to the deep oceans. $M_{pre}$, $\eta$, and $\delta_M$ are given as 590 billion tons of carbon equivalent, .64, and .0833 per decade, respectively.

As greenhouse gases accumulate, the amount of radiation near the Earth's surface increases. This relationship is:

$$F(t) = 4.1\{\log[M(t)/590]/\log(2)\} + O(t) \tag{A7}$$

where $F(t)$ is radiative forcing in watts per square meter and $O(t)$ is exogenous forcings from other greenhouse gases (primarily $CH_4$ and $N_2O$). $F(t)$

is then related to a global average surface temperature increase $\Delta T(t)$ via equations which describe the heat transfer between the atmosphere, upper oceans, and deep oceans. One assumption of these equations is that a doubling of $CO_2$ levels will lead to a 3°C warming — a quantity know as the "climate sensitivity"; variations in this important parameter are easy to incorporate into DICE. Further details regarding the relationship between $F(t)$ and $\Delta T(t)$ can be found in [Schneider and Thompson, 1981].

It is through $\Delta T(t)$ that the climate side of the DICE model provides feedback to the economic side. Specifically, using temperature as an indicator for climatic change, damage from climatic change is given by:

$$d(t) = \alpha_1 \Delta T(t)^{\alpha_2} \tag{A8}$$

where $d(t)$ is in fractional loss of global output, and $\alpha_1$ and $\alpha_2$ are estimated as .00148 and 2 respectively.

The amount of temperature increase, and hence the amount of damage from climatic change, can be influenced by choosing $\mu(t)$. However, controlling emissions also carries a cost:

$$TC(t) = \beta_1 \mu(t)^{\beta_2} \tag{A9}$$

where $TC(t)$ is in fractional loss of global output, and $\beta_1$ and $\beta_2$ given by Nordhaus as .0686 and 2.887, respectively. This implies that a small ($< 10\%$) reduction in emissions can be achieved with relatively low cost, but drastic cuts are fairly expensive (e.g., a 50% cut is emissions would cost about 1% of global output). Although these values are debatable (e.g., [Repetto and Austin, 1997]), we use the original DICE formulation in order to focus on the sensitivity of policy options to alternative damage functions.

These two costs are combined in the $\Omega(t)$ term:

$$\Omega(t) = \frac{1 - TC(t)}{1 + d(t)} \tag{A10}$$

$\Omega(t)$ does not fully capture the damages of the $d(t)$ term, since $\frac{1}{1+d(t)}$ is a reasonable approximation of $1 - d(t)$ only for small values of $d(t)$. In the extreme case, with $d(t) = 1$, the world "only" experiences a 50% reduction in output due to damage from climatic change, rather than a complete loss of global output. However, since the original model is being used here to analyze the consequences of uncertainties in damage functions and to display methods to present uncertainties quantitatively, we do not reformulate $\Omega(t)$ in this study.

This completes the linking of the economic and climatic sides of the DICE model.

# B Deriving Damage Functions from Damage Distributions

In this appendix, we describe the derivation of the damage functions used in our probabilistic analysis of the DICE model.

Given positive damage estimates from the scenario A and scenario C damage distributions (call the estimates $y_3$ and $y_6$, respectively), the corresponding damage function is given by:

$$d(t) = a\Delta T(t)^b \tag{B1}$$

where:

$$a = y_6/6^b \tag{B2}$$

$$b = \frac{\log(y_6/y_3)}{\log 2} \tag{B3}$$

About 10% of the time, however, at least one of the damage estimates is negative — that is, a net benefit from climatic change is predicted. In this case, there is no longer a function of the form $ax^b$ which contains the three damage estimates (0, $y_3$, and $y_6$). However, the three estimates can be described by a function with the form $a(x+c)^b+d$ (roughly, a translated parabola). We cannot, however, solve for all four variables $(a, b, c, d)$ with only three data points. As a result, we reassert the assumption that damage is a quadratic function of temperature increase (i.e., we fix $b = 2$). Now we can derive a damage function from $y_3$ and $y_6$ as follows:

$$d(t) = a(\Delta T(t) + c)^2 + d \tag{B4}$$

where:

$$a = \frac{y_6 - 2y_3}{18} \tag{B5}$$

$$c = \frac{y_3 - 9a}{6a} \tag{B6}$$

$$d = -ac^2 \tag{B7}$$

Table 1: A comparison of IPCC damage estimates for a $CO_2$-doubling scenario (damage is for U.S. only). Both temperature increase and the corresponding amount of damage are estimated. *Source:* [Bruce et al., 1996].

| Researcher | Warming ($^\circ C$) | Damage (% of GDP) |
|---|---|---|
| Cline | 2.5 | 1.1 |
| Fankhauser | 2.5 | 1.3 |
| Nordhaus | 3.0 | 1.0 |
| Titus | 4.0 | 2.5 |
| Tol | 2.5 | 1.5 |

Table 2: Expert Opinion on Climate Change (in %GWP loss). *Source:* [Nordhaus, 1993].

| Respondent | Scenario A | | | Scenario C | | |
|---|---|---|---|---|---|---|
| Number | 10 %ile | 50 %ile | 90 %ile | 10 %ile | 50 %ile | 90 %ile |
| 1 | 0.7 | 1.3 | 8.8 | 1.4 | 2.6 | 14.1 |
| 2 | -0.3 | 1.3 | 6.0 | 2.0 | 3.8 | 15.0 |
| 3 | 0.1 | 0.3 | 0.6 | 0.2 | 0.8 | 3.0 |
| 4 | -0.5 | 1.5 | 5.0 | -0.5 | 4.0 | 9.0 |
| 5 | 3.3 | 16.3 | 31.3 | 6.5 | 30.0 | 62.5 |
| 6 | 1.2 | 1.9 | 3.6 | 3.0 | 6.0 | 18.0 |
| 7 | 0.0 | 2.5 | 6.0 | 2.0 | 5.0 | 15.0 |
| 8 | 10.0 | 21.0 | 30.0 | 20.0 | 62.0 | 100.0 |
| 9 | -1.0 | 1.5 | 8.0 | 0.0 | 4.0 | 15.0 |
| 10 | 1.0 | 5.0 | 14.0 | 4.0 | 15.0 | 30.0 |
| 11 | 0.8 | 1.8 | 5.0 | 2.8 | 6.4 | 17.0 |
| 12 | 1.0 | 2.0 | 5.0 | 3.0 | 6.0 | 15.0 |
| 13 | -1.0 | 3.0 | 8.0 | 1.0 | 6.0 | 15.0 |
| 14 | 0.0 | 0.0 | 0.8 | 0.0 | 2.0 | 5.0 |
| 15 | -2.0 | 2.0 | 6.0 | 3.0 | 10.0 | 17.0 |
| 16 | -0.5 | 0.5 | 1.0 | 2.0 | 3.5 | 5.0 |
| 17 | -0.5 | 0.3 | 0.5 | -1.0 | 1.0 | 5.0 |
| 18 | 0.0 | 2.0 | 4.0 | 10.0 | 20.0 | 30.0 |
| Mean | 0.7 | 3.6 | 8.0 | 3.3 | 10.4 | 21.7 |
| Nat. Sci. | 5.5 | 13.0 | 22.0 | 12.0 | 38.5 | 65.0 |
| Env. Econ. | 1.2 | 6.6 | 14.3 | 3.2 | 13.3 | 31.8 |
| Other | -0.1 | 1.5 | 4.4 | 2.0 | 3.9 | 12.7 |

Table 3: Comparison of Monte Carlo simulation results with the standard DICE model. "Surprise" values are 95th percentile results. Explicitly including low probability, high consequence outcomes alerts policy makers to consider strategic hedging options to reduce the risk of experiencing catastrophic outlier events.

| Source of | Optimal Carbon Tax ($/ton C) | | |
|---|---|---|---|
| Data | 1995 | 2055 | 2105 |
| DICE | 5.24 | 15.04 | 21.73 |
| Median | 22.85 | 51.72 | 66.98 |
| Mean | 40.42 | 84.10 | 109.73 |
| "Surprise" | 193.29 | 383.39 | 517.09 |

# Figure Legends

Figure 1: The DICE model reformulated with damage functions derived from damage estimates by Cline (C), Fankhauser (F), Nordhaus (N), Titus (Ti), and Tol (To). The original model used the damage function derived from Nordhaus's personal damage estimates (N). Figure 1a displays the damage functions derived from published IPCC damage estimates [Bruce et al., 1996]. Figure 1b shows the loss of discounted consumption in the BAU scenario for each of the damage functions in Figure 1a (where discounted consumption is all consumption occurring after 1989, discounted to 1990 by the rate of interest on goods and services calculated in the standard optimal DICE run). In essence, these curves represent the damage of unmitigated climate change. Of course, the curves shown here represent only a small fraction of overall consumption (the largest difference between the highest and lowest curves is less than 1% of total discounted consumption). Figure 1c gives optimal carbon tax levels corresponding to each of the damage functions in Figure 1a.

Figure 2: Best estimates of survey respondents for total climate damage plotted against respondents' predictions for the percentage of damage occurring in the standard national accounts. Figure 2a presents results for Scenario A (a 3°C warming by 2090), and Figure 2b shows estimates for Scenario C (a 6°C warming by 2090). Several respondents did not complete this portion of the survey. Note that there is a strong suggestion that respondents estimating large climate damages are more likely to assign a large proportion of such damages to non-market categories.

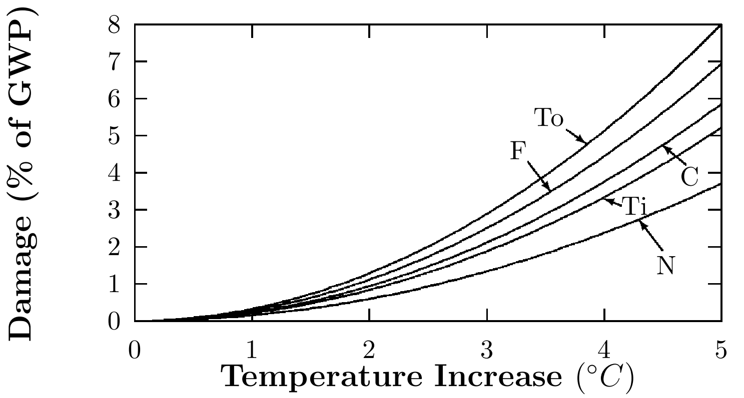
Figure 3: The DICE model reformulated with damage functions derived from damage estimates given by experts. Figure 3a shows the "disciplinary damage functions" derived from an expert survey [Nordhaus, 1994a], for natural

scientists ("Nat. Sci."), environmental economists ("Env. Econ."), and other social scientists ("Soc. Sci."), primarily conventional economists. The original DICE damage function ("DICE '92") is also shown for comparison. In Figures 3b and 3c, optimal policy given the median damage estimates of natural scientists ($\times$) is compared with optimal policies with the high (90th percentile) damage estimates of all of the experts ($\Box$), the median damage estimates of all of the experts ($+$), and with the damage estimates used in the original DICE model ($\Diamond$). Figure 3b gives optimal carbon tax levels for each group of damage estimates, and Figure 3c displays the corresponding optimal emission control rates. The increases in global average temperature by 2105 associated with these policies are 2.77°C, 2.94°C, 3.10°C, and 3.20°C, respectively, suggesting that even the largest control rate abates only a modest fraction of the projected climate changes. In addition, Figure 3c shows optimal control rates with the median damage estimates of all experts and a 1.5% social rate of time preference ($\triangle$)—half the value used for ($+$).

Figure 4: "Aggregate expert damage distributions" for warming scenarios A (3° by 2090) and C (6° by 2090). These distributions are used to derive "randomly sampled" damage functions for use in a probabilistic analysis of the DICE model under uncertainty. Figures 4a and 4b show the cumulative distribution functions and probability density functions, respectively, of the damage distributions. Figure 4c displays several example damage functions used in the Monte Carlo simulation. The "50%ile" damage function (for example) is the function through all of the following: the origin (since we assume zero damage with no temperature increase), the median of the damage distribution for scenario A at $\Delta T(t) = 3$°C, and the median of the damage distribution for scenario C at $\Delta T(t) = 6$°C.

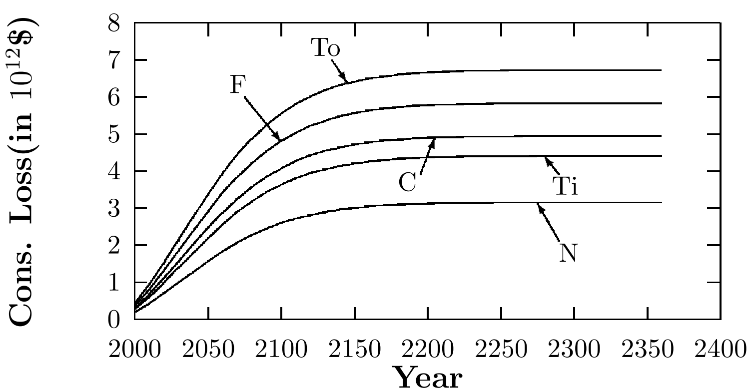
Figure 5: Results of the Monte Carlo simulation based on surveyed experts

[Nordhaus, 1994a], presented as distributions of optimal carbon tax levels. Figures 5a and 5b give the cumulative distribution function, $F(x)$, and probability density function, $f(x)$, respectively, for optimal taxes in 1995. Figures 5c and 5d show similar functions for optimal taxes in 2055; Figures 5e and 5f display the corresponding functions for 2105.



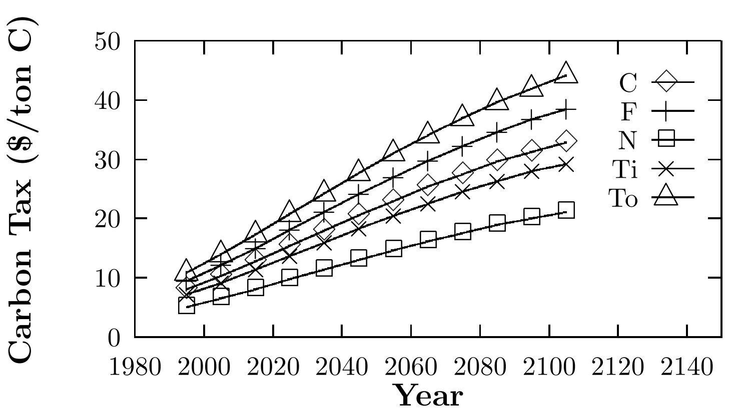
Damage (% of GWP)
Temperature Increase (°C)
To
F
C
Ti
N
0
1
2
3
4
5
6
7
8


(a)



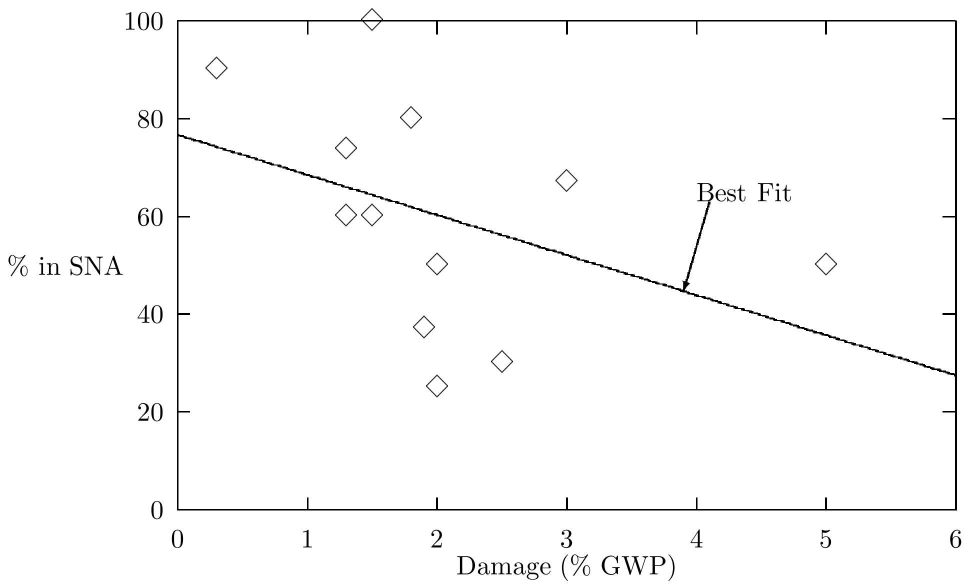
Cons. Loss(in $10^{12}$\$)
Year
To
F
C
Ti
N
2000
2050
2100
2150
2200
2250
2300
2350
2400


(b)



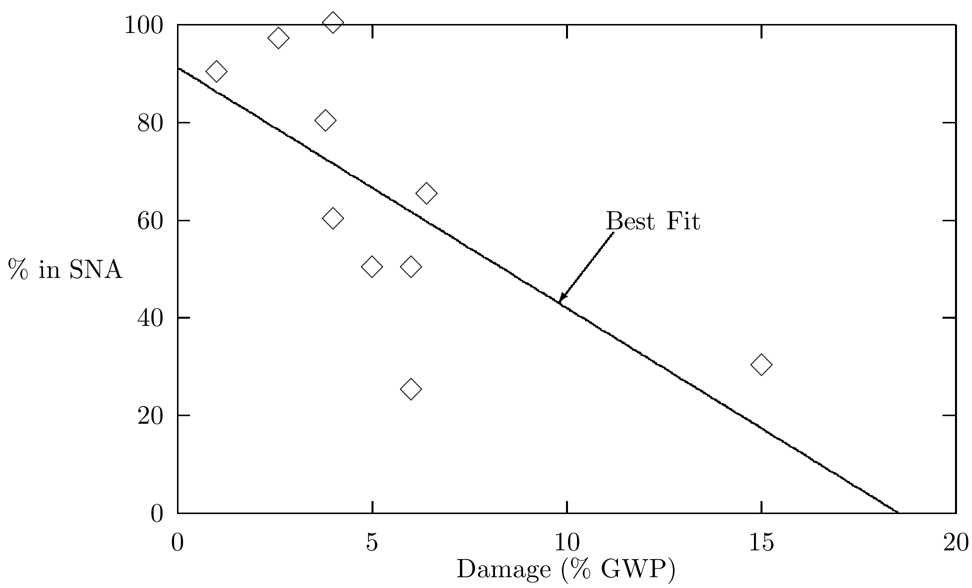
Carbon Tax (\$/ton C)
Year
C
F
N
Ti
To
1980
2000
2020
2040
2060
2080
2100
2120
2140


(c)



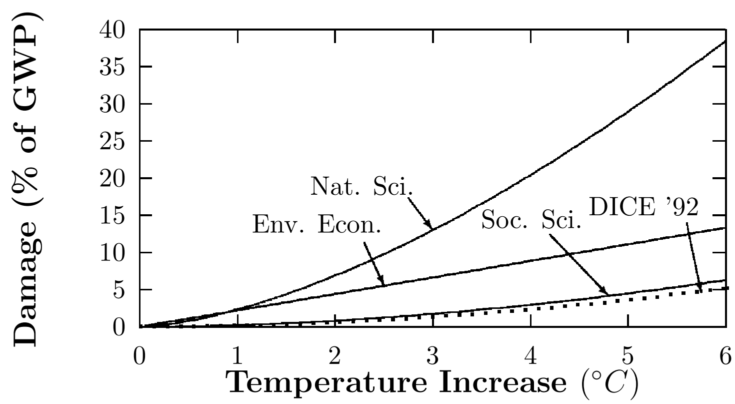
100
80
60
40
20
0
% in SNA
Best Fit
0
1
2
3
4
5
6
Damage (% GWP)


(a)

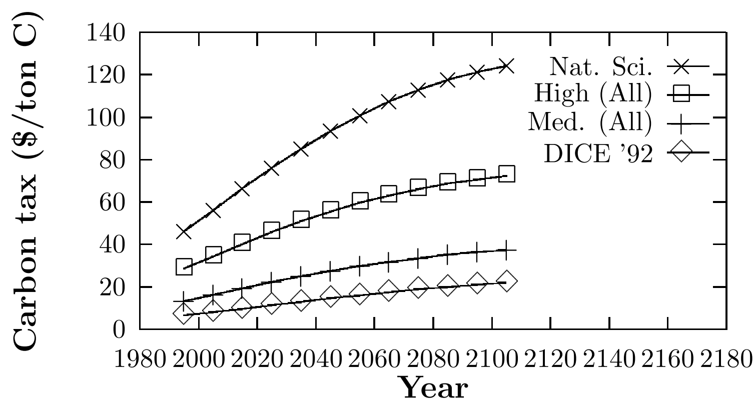


100
80
60
40
20
0
% in SNA
Best Fit
0
5
10
15
20
Damage (% GWP)

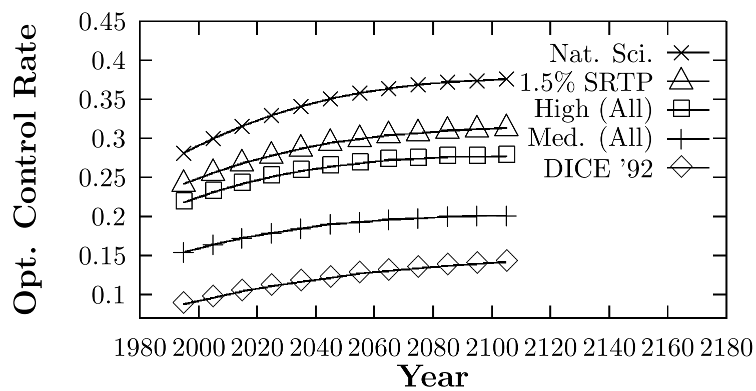

(b)


Damage (% of GWP)
40
35
30
25
20
15
10
5
0
Nat. Sci.
Env. Econ.
Soc. Sci.
DICE '92
0
1
2
3
4
5
6
Temperature Increase (°C)

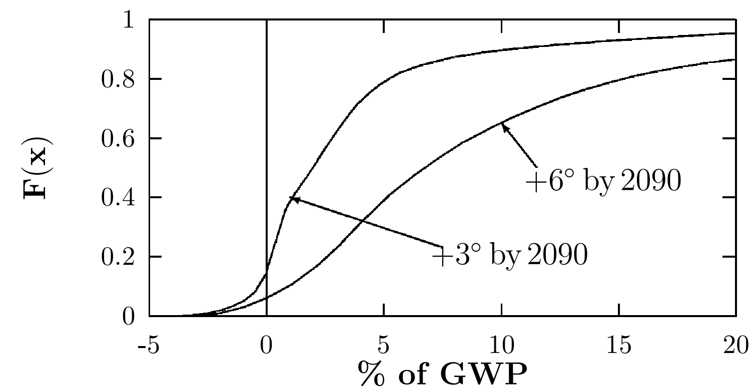
(a)


Carbon tax ($/ton C)
140
120
100
80
60
40
20
0
Nat. Sci.
High (All)
Med. (All)
DICE '92
1980 2000 2020 2040 2060 2080 2100 2120 2140 2160 2180
Year

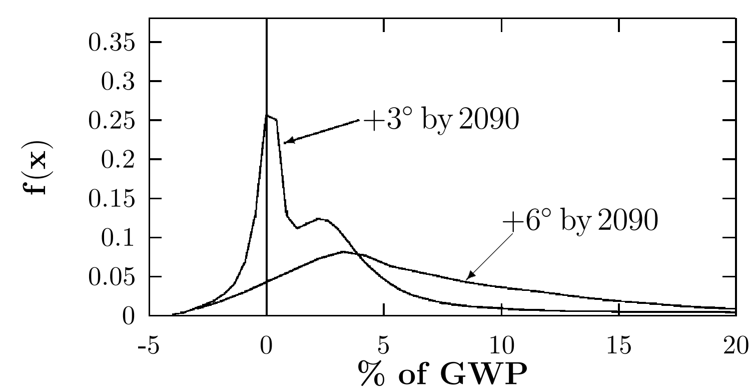
(b)


Opt. Control Rate
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
Nat. Sci.
1.5% SRTP
High (All)
Med. (All)
DICE '92
1980 2000 2020 2040 2060 2080 2100 2120 2140 2160 2180
Year

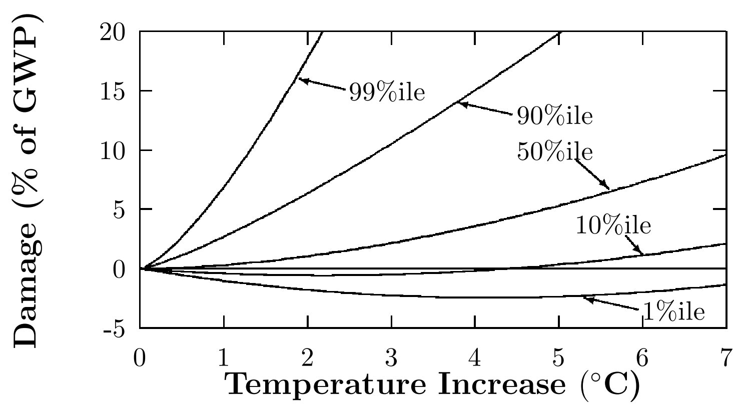
(c)


F(x)
% of GWP
+6° by 2090
+3° by 2090

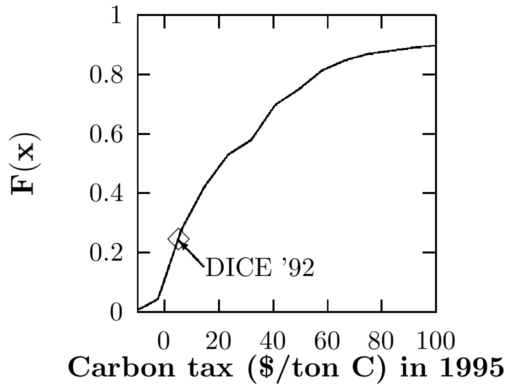
(a)


f(x)
% of GWP
+3° by 2090
+6° by 2090

(b)

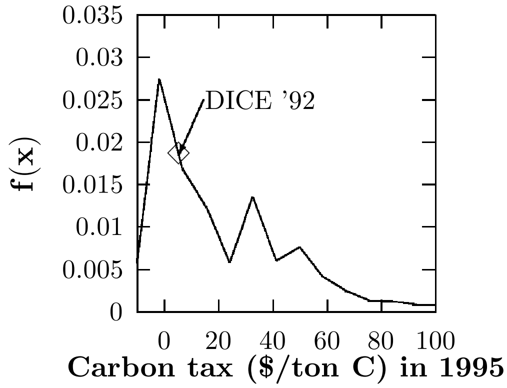

Damage (% of GWP)
Temperature Increase (°C)
99%ile
90%ile
50%ile
10%ile
1%ile

(c)



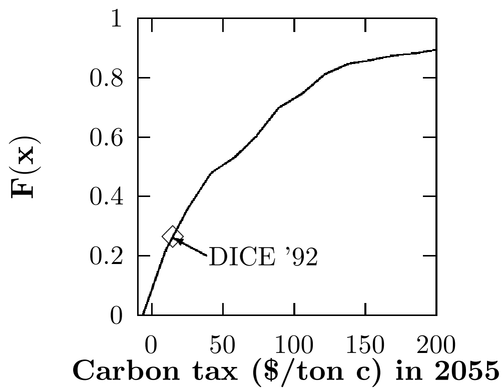
F(x)
1
0.8
0.6
0.4
0.2
0
DICE '92
0 20 40 60 80 100
Carbon tax ($/ton C) in 1995


(a)



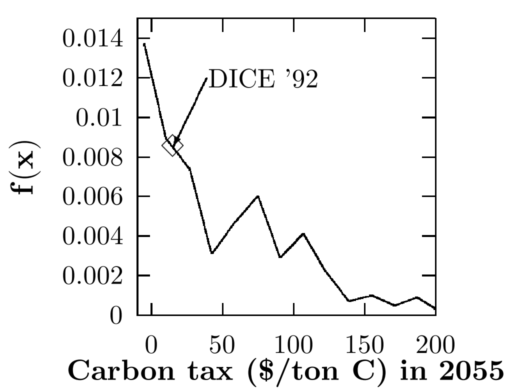
f(x)
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
DICE '92
0 20 40 60 80 100
Carbon tax ($/ton C) in 1995


(b)



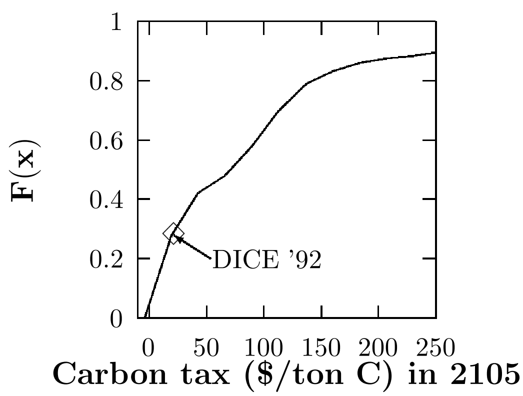
F(x)
1
0.8
0.6
0.4
0.2
0
DICE '92
0 50 100 150 200
Carbon tax ($/ton c) in 2055


(c)



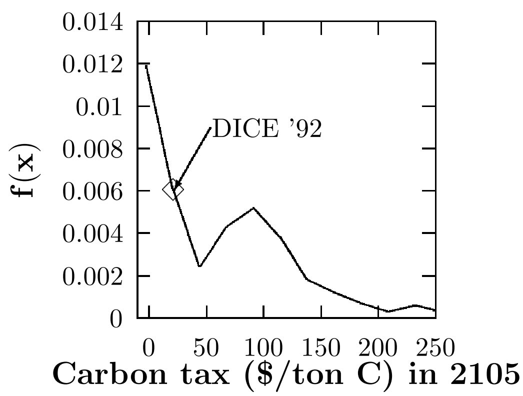
f(x)
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
DICE '92
0 50 100 150 200
Carbon tax ($/ton C) in 2055


(d)



F(x)
1
0.8
0.6
0.4
0.2
0
DICE '92
0 50 100 150 200 250
Carbon tax ($/ton C) in 2105


(e)



f(x)
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
DICE '92
0 50 100 150 200 250
Carbon tax ($/ton C) in 2105


(f)