1/ New paper "Automated Market Making and Arbitrage Profits in the Presence of Fees" by @jason_of_cs @ciamac @Tim_Roughgarden
https://moallemi.com/ciamac/papers/lvr-fee-model-2023.pdf

Automated Market Making and Arbitrage Profits in the Presence of Fees

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Abstract

We consider the impact of trading fees on the profits of arbitrageurs trading against an automated marker marker (AMM) or, equivalently, on the adverse selection incurred by liquidity providers due to arbitrage. We extend the model of Milionis et al. [2022] for a general class of two asset AMMs to both introduce fees and discrete Poisson block generation times. In our setting, we are able to compute the expected instantaneous rate of arbitrage profit in closed form. When the fees are low, in the fast block asymptotic regime, the impact of fees takes a particularly simple form: fees simply scale down arbitrage profits by the fraction of time that an arriving arbitrageur finds a profitable trade.

2/ The goal is to understand the impact of fees on arbitrage trading against AMMs, and use this as quantitative guidance to understand how to set fees and how to design AMMs to minimize the MEV extracted by arbs and tradeoffs therein.

3/ The starting point is LVR, i.e., how much do DEX LPs lose to DEX-CEX arb in an idealized setting, trading in continuous time (no discrete blocks) and with no trading fees
https://moallemi.com/ciamac/papers/lvr-2022.pdf

4/ What happens when we incorporate discrete block generation and trading fees? Both are frictions that impact arbitrage trade. Fees create a “no-trade” region, where although the DEX and CEX prices differ, the mispricing does not exceed the fee and hence arbs don’t trade.

5/ Under the assumption of Poisson block generation, our first result is to solve for the steady state distribution of the DEX-CEX mispricing, which follows a jump diffusion process. This allows us to quantify the probability of the no-trade region.

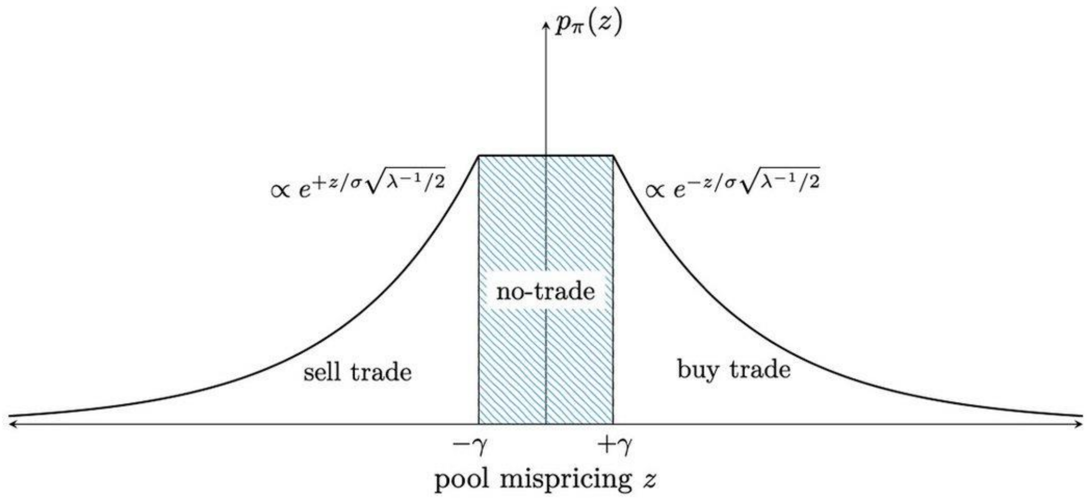


Figure 3: The density $p_\pi(z)$ of the stationary distribution $\pi(\cdot)$ of mispricing z, illustrating trade and no-trade regions for an arbitrageur.

6/ We show that, if fees are gamma and mean interblock time is Delta t, the probability that a block contains a trade (the probability of being outside the no trade region) takes a simple form:

$$P_{\text{trade}} = \frac{1}{1 + \gamma/(\sigma\sqrt{\Delta t/2})}$$

7/ This probability depends on the fee measured in units of typical return (stdev) over half the interblock time. When fees are high or the interblock time is low, it becomes less likely that arbs can profit on any given block. For example:

$\Delta t \setminus \gamma$	1 bp	5 bp	10 bp	30 bp	100 bp
10 min	96.7%	85.5%	74.7%	49.6%	22.8%
2 min	92.9%	72.5%	56.9%	30.5%	11.6%
12 sec	80.7%	45.6%	29.5%	12.3%	4.0%
2 sec	63.0%	25.4%	14.5%	5.4%	1.7%
50 msec	21.2%	5.1%	2.6%	0.9%	0.3%

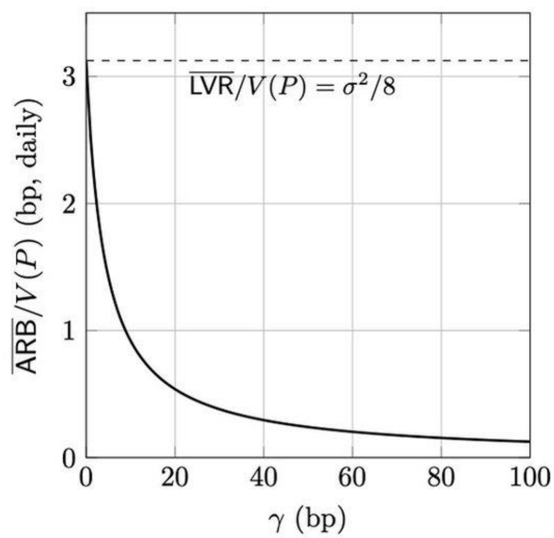
Table 1: The probability of trade $\mathsf{P}_{\mathsf{trade}}$, or, equivalently, the fraction of blocks with contain an arbitrage trade, given asset price volatility $\sigma = 5\%$ (daily), with varying mean interblock times $\Delta t \triangleq \lambda^{-1}$ and fee levels γ (in basis points).

8/ Our main result is to compute arb profits in closed form for general CFMMs.

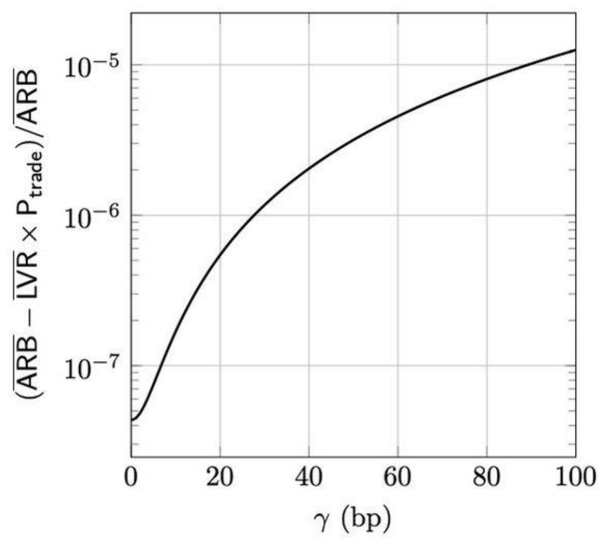
9/ The formulas simplify when fees are low and blocks are frequent (the “fast block” regime), in this case arb profits are simply LVR scaled down by the probability of trade.

$$\overline{\mathsf{ARB}} \approx \overline{\mathsf{LVR}} \times \mathsf{P}_{\mathsf{trade}}.$$

10/ This approximation is very accurate for typical parameter values.



(a) The normalized intensity of arbitrage profit $\overline{\mathsf{ARB}}/V(P)$ as a function of the fee γ.



(b) The relative error of the approximation (10), i.e., $(\overline{\mathsf{ARB}} - \overline{\mathsf{LVR}} \times \mathsf{P}_{\mathsf{trade}})/\overline{\mathsf{ARB}}$, as a function of the fee γ.

Figure 5: The constant product market maker case, with $\sigma = 5\%$ (daily) and mean interblock time $\Delta t \triangleq \lambda^{-1} = 12$ (seconds).

11/ Note that there is an interesting discontinuity here: when fees are zero, arb profits are basically LVR — they do not vary much with the interblock time.

12/ On the other hand, once fees are even slightly positive, arb profits scale with sqrt(interblock time) and shrink to zero with faster blocks.

13/ This is consistent with the observations of many (e.g., @0x94305 @MaxResnick1) that faster blocks are an easy way to mitigate DEX MEV, perhaps at the cost of reducing decentralization.

14/ We also observe that, in the fast block regime,

(arb profits net of fees) + (fees paid by arbs to the pool) ≈ LVR

15/ Though LVR was developed assuming no fees and continuous trading, even with fees and discrete blocks, LVR is roughly the profit gross of fees of arbing the pool. Introducing fees simply changes how LVR is split and who earns it (arbs or pool LPs).

16/ This split is precisely quantified by our model.

17/ One way to think about the choice of fee is through the framing of @rithvikra0 and @theshah39: fees create a tradeoff between losing money to arbs and the accuracy of the pool prices.

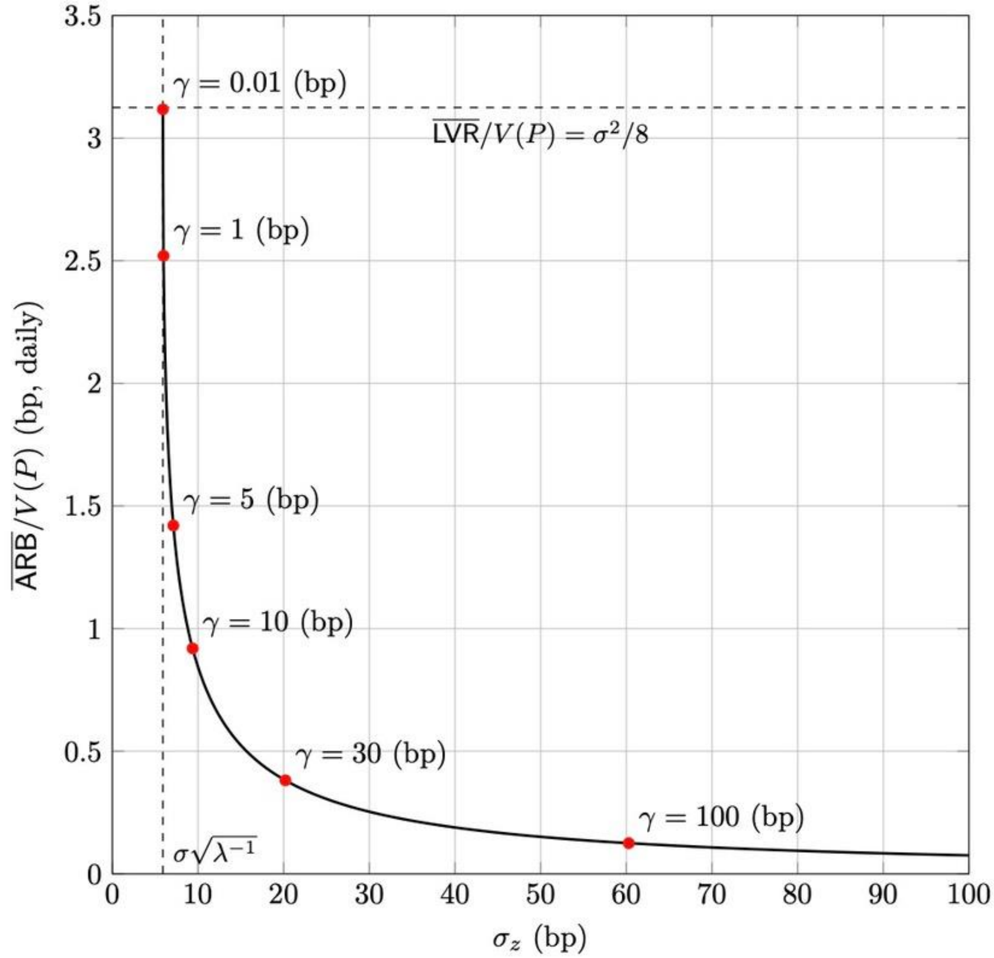


Figure 6: Efficient frontier between mispricing error and arbitrage profits for different choices of fees, for a constant product market maker. Here, we set $\sigma = 5\%$ (daily) and $\lambda^{-1} = 12$ (seconds). The horizontal axis is the standard deviation of the steady state pool mispricing σ_z. The vertical axis is the intensity per unit time of arbitrage profits per dollar value of the pool $\overline{\mathsf{ARB}}/V(P)$.

18/ h/t to @0x94305, who has worked on similar results