CS364B: Exercise Set #5

Optional (no due date)

Lecture 9 Exercises

Exercise 28 (Optional – Do Not Hand In)

Let $\mathbf{x}$ be a maximal-in-distributional-range (MIDR) allocation rule. Recall that coupling $\mathbf{x}$ with the payment rule

$$p_i(\mathbf{v}) = \max_{D \in \mathcal{D}} \mathbf{E}_{\omega \in D}\left[\sum_{k \neq i} v_i(\omega)\right] \mathbf{E}_{\omega \in D^*}\left[\sum_{k \neq i} v_i(\omega)\right] \tag{1}$$

yields a DSIC mechanism, assuming that all players act to maximize their expected quasi-linear utility.

The definition in (1) is worrisome from a computational point of view, since expectations over exponentially large sample spaces are not always tractable to compute. Prove that, for the specific mechanism that we described for scenario #8, this payment rule can be computed in polynomial time.

Exercise 29 (Optional – Do Not Hand In)

With risk-neutral bidders, we can equally well couple an MIDR allocation rule $\mathbf{x}$ with a randomized payment rule, provided the expected payment $\mathbf{E}[p_i(\mathbf{v})]$ equals the right-hand side of (1). Give an implementation of such a payment rule $\mathbf{p}$ via a reduction to $n+1$ invocations of the allocation rule $\mathbf{x}$, where n is the number of bidders.

Exercise 30 (Optional – Do Not Hand In)

A randomized mechanism is *ex post individually rational (EPIR)* if the utility of a truthtelling bidder is non-negative with probability 1. Are the payment rules in the previous two exercises generally EPIR?

Exercise 31 (Optional – Do Not Hand In)

For the specific mechanism that we described for scenario #8, exhibit a randomized polynomial-time payment rule that satisfies (1) in expectation and also yields an EPIR mechanism.

[Hint: use payments that are proportional to valuations.]

Lecture 10 Exercises

Exercise 32 (Optional – Do Not Hand In)

Consider a coverage valuation v on the item set $U = \{1, 2, \ldots, m\}$, given explicitly as subsets $A_1, \ldots, A_m$ of a ground set X. Recall that, by definition,

$$v(S) = |\cup_{j \in S} A_j| \,.$$

Prove that answering a demand query — given a price vector $\mathbf{p}$ on items, compute a subset of $\operatorname{argmax}_{S \subseteq U}\{v(S) - \sum_{j \in S} p(j)\}$ — is an NP-hard problem.

[Hint: reduce from Set Cover.]

Exercise 33 (Optional – Do Not Hand In)

In lecture we only worked with *unweighted* coverage functions. More generally, a coverage function is defined by nonnegative weights w_a on elements a of a ground set X and subsets $A_1, \ldots, A_m$ corresponding to the items of U. The valuation is defined as

$$v(S) = \sum_{a \in \cup_{j \in S} A_{ij}} w_a.$$

What changes to the mechanism and its analysis described in lecture are needed to extend the DSIC $(1 - \frac{1}{e})$-approximation to weighted coverage functions?